Nonequilibrium turbulence research: The physical processes and their potential relevance to wave turbulence phenomena

Summary of major strands of research presented at the inaugural *Nonequilibrium Turbulence* UKFN SIG Meeting, focusing on the classical side of the SIG

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Talk Outline

1. New findings in superfluid turbulence
2. Equilibrium turbulence
3. Non-equilibrium dissipation in grid and wake turbulence
4. Non-equilibrium dissipation as a consequence of phase lag behaviour
5. Macroscale coupling in boundary-layers.

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Recent Summarial Works

At our SIG, Joe Vinen FRS provided a fascinating overview of ~50 years of work on superfluid turbulence. Additional recent works we have authored:

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Dissipation in Turbulent Flows

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Preface

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JSPS Supported Symposium on Interscale Transfers and Flow Topology in Equilibrium and Non-equilibrium Turbulence (Sheffield, UK, September 2014)

SCIENTIFIC REPORTS

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Regimes of turbulence without an energy cascade

C. F. Barenghi[1], Y. A. Sergeev[2] & A. W. Baggaley[1]

PRL **118,** 135301 (2017) PHYSICAL REVIEW LETTERS week ending 31 MARCH 2017

Superfluid Boundary Layer

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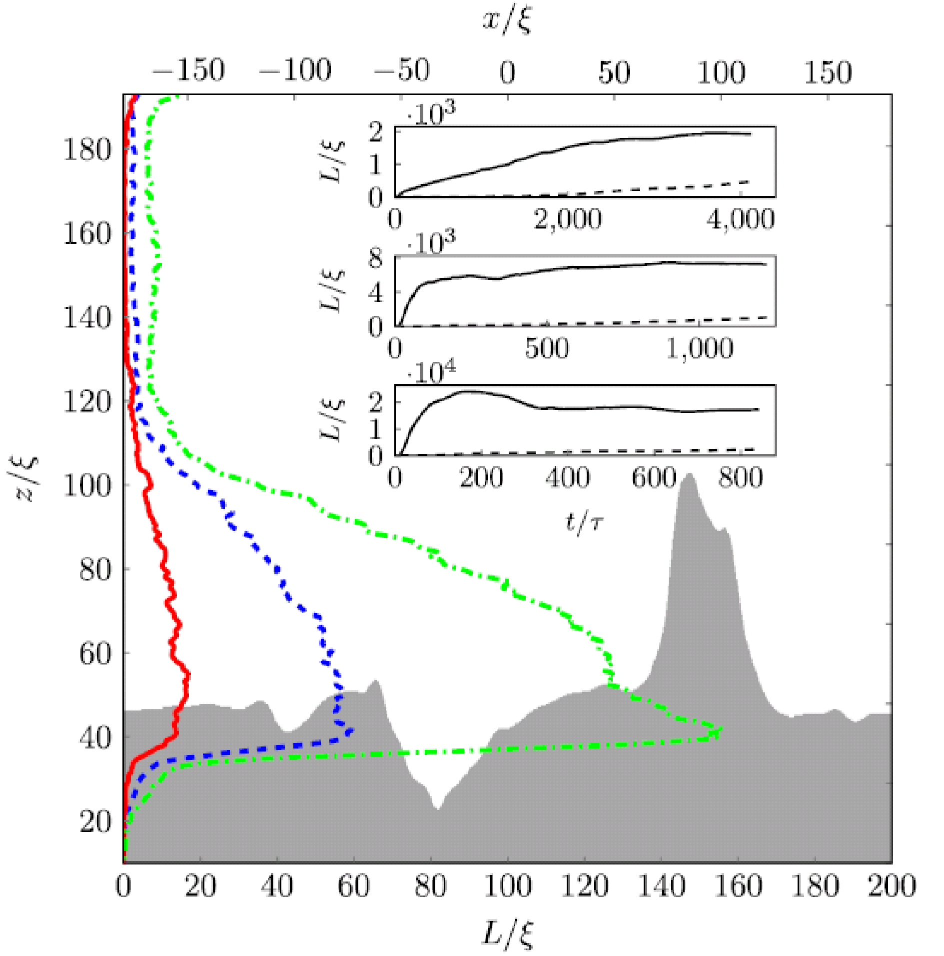
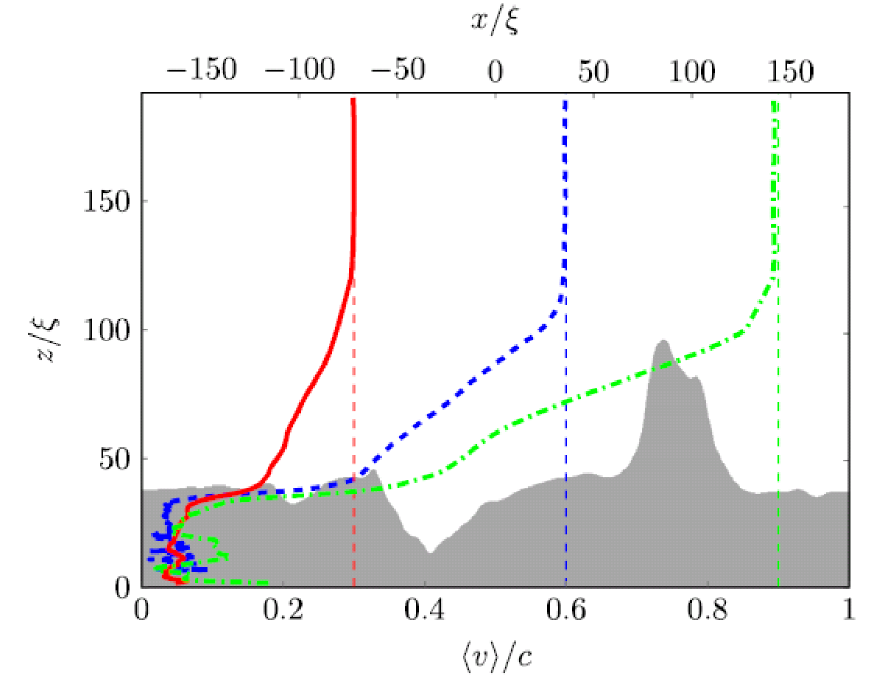




Equilibrium Ideas

- Scale separation between scale at which energy is input and scales at which it is dissipated.
- Thus, the scales of velocity and the scales of velocity derivatives are independent.
- Because small scales are not coupled to the large, and evolve fast compared to these scales, isotropic, equilibrium behaviour at small scales can be postulated.
- We move from large to small scales via a Richardson cascade, which is an energy transfer process with dissipation only acting on the smallest scales. This cascade tends to be prioritised in one's turbulence formalism

Equilibrium Ideas

Models for closing the Navier-Stokes equations (whether one point closures for RANS or those for eddy-resolving models) draw upon such principles.

The imposition of an instantaneous balance between production and dissipation is an assumption in classical closures.

"What part of modeling is in serious need of work? Foremost, I would say, is the mechanism that sets the level of dissipation in a turbulent flow, particularly in changing circumstances."

Lumley (1992)

Equilibrium Ideas

Given a length scale, *r*, and an associated velocity, $u(r)$, kinetic energy transfer across this scale can be estimated dimensionally as $u(r)^3 / r$.

The rate will be set at the top of the cascade by the integral length and velocity scales, ℓ and U

In equilibrium, this transfer to dissipative scales, will be balanced by the dissipation rate:

$$\epsilon = \nu \langle \mathsf{S}^2 \rangle$$

Hence,

$$\epsilon \sim \frac{U^3}{\ell}$$

where typically, ℓ is estimated from the longitudinal integral scale and $U = \sqrt{3/2}\,u_1'$ (Batchelor, 1953)

Equilibrium Ideas

This expression for the dissipation leads to a universal (Reynolds number independent) dissipation constant at high enough Reynolds number:

$$\epsilon = C_\epsilon \frac{U^3}{\ell}$$

According to Tennekes and Lumley (1972), this "should not be passed over lightly. It is one of the cornerstone assumptions of turbulence theory."

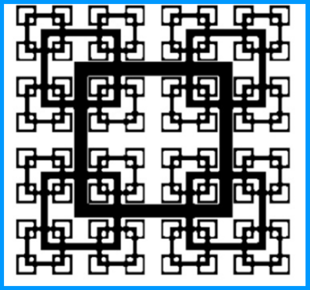
Interestingly, Taylor (1935) says that C_ϵ is a constant for geometrically similar boundaries.

Fractal grid experiments

Initially conceived of to violate the assumptions in classical forcing:
Multiple forcings at a range of scales, dropping away in a power-law fashion.

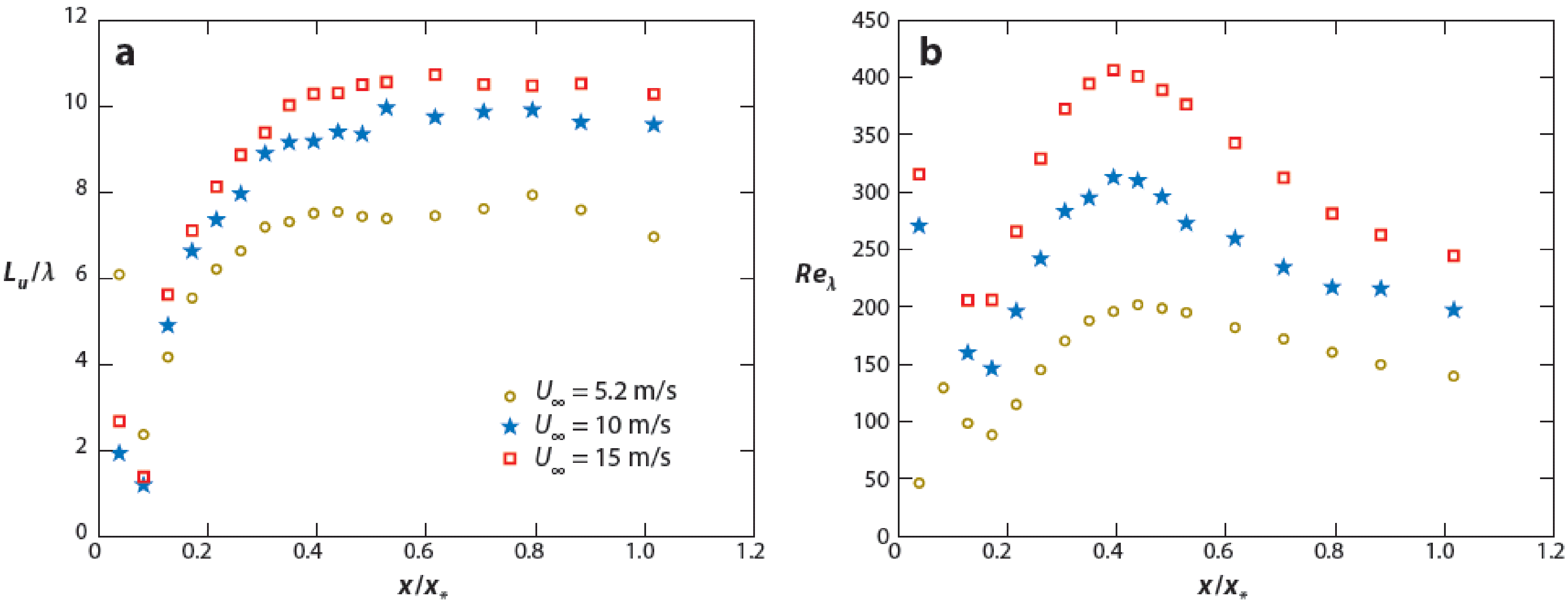
A consequence of this for decaying grid turbulence was that a region was extended in size that previous work had tended to ignore – that near the peak in turbulence intensity.

Studies in this region have found very different, non-equlibrium behaviour. (Seoud and Vassilicos, 2007; Mazellier and Vassilicos, 2010).



Subsequently, this has been shown to be true in regular grids

Fractal grid experiments



x_* is a wake interaction length scale, used to predict where the turbulence intensity peaks, which occurs at $x / x_* = 0.4$.

The implication is that beyond x_{peak} there is a different type of behaviour.

Vassilicos, *Ann. Rev. Fluid Mech*., 2015.

Fractal grid experiments

Traditional experiments either:

(a) Fix a position *x* and then vary the Reynolds number for the experiment, Re_m

(b) Fix the Re_m and vary *x*.

As *x* changes, so does the turbulence intensity, implying two Reynolds numbers are relevant:

$$\mathrm{Re}_m = \frac{U_\infty M}{\nu}$$

$$\mathrm{Re}_\ell = \frac{u_1' \ell}{\nu}$$

A macro-Reynolds number with a length scale, *m*, set by the forcing and a local Reynolds number based on observed integral and rms velocity at a point.

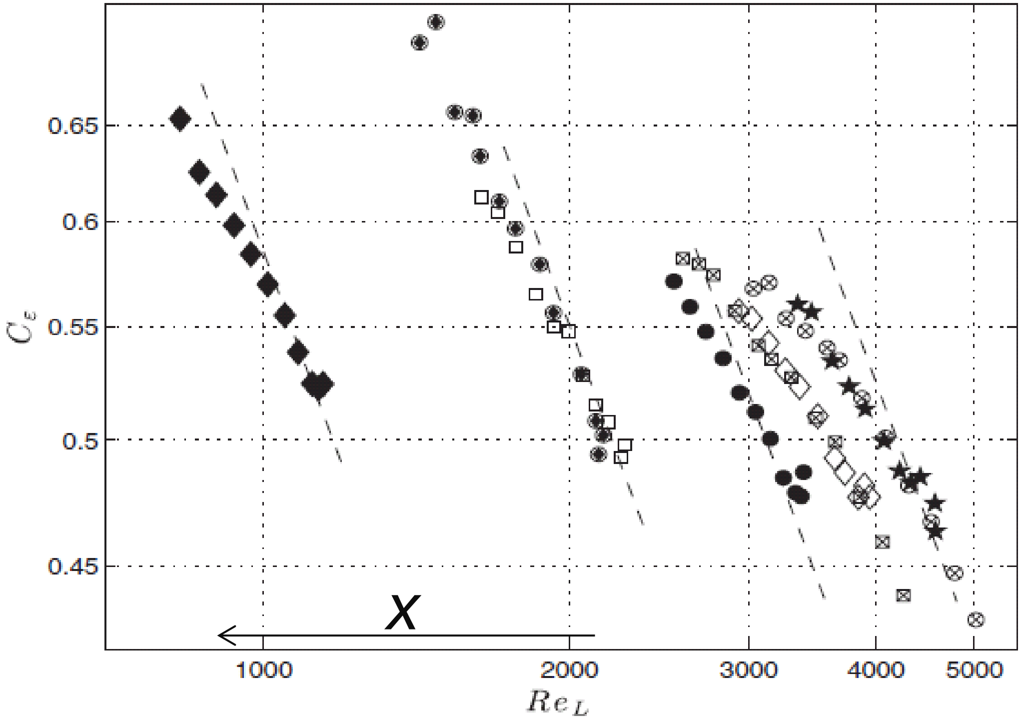
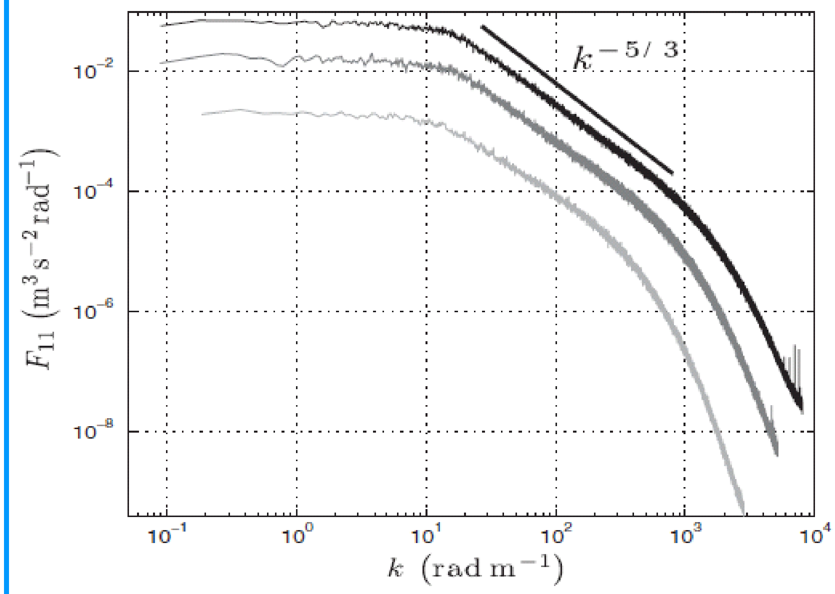
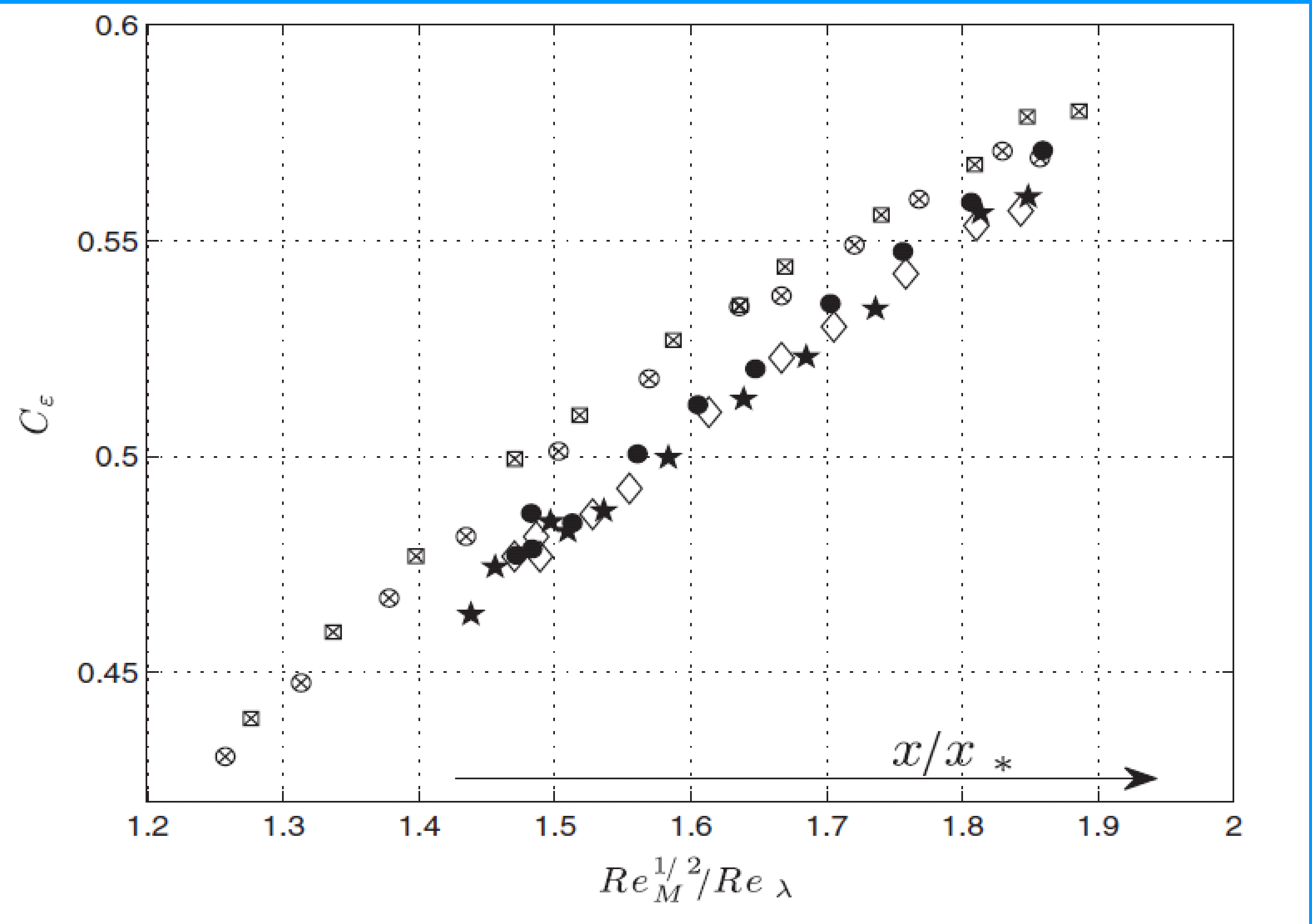


FIG. 3. Normalized energy dissipation C_ε versus local Reynolds number Re_L of turbulence generated by FSG, RG230 and RG115 for different inflow Reynolds numbers Re_M. The dashed lines follow $\propto \mathrm{Re}_L^{-1}$ for different Re_M. The Re_λ values of the data in this plot range between 140 and 418.



Valente and Vassilicos (2012), *Phys. Rev. Lett.* 108, 214503



$$C_\varepsilon \propto \frac{\mathrm{Re}_M}{\mathrm{Re}_L} \propto \frac{\mathrm{Re}_M^{1/2}}{\mathrm{Re}_\lambda}$$

Valente and Vassilicos (2012), *Phys. Rev. Lett.* 108, 214503

Implications for e.g. wake scaling

- Wake development proceeds with a wake width, δ, taken to define the integral scale and a velocity scale given by the velocity deficit on the centre-line, u_0.
- $C_\epsilon = \text{const.}$ The Kolmogorov-Richardson scaling

then implies the traditional scalings for wake evolution:

$$u_0/U_\infty \sim ((x-x_0)/\theta)^{-2/3} \qquad \delta/\theta \sim ((x-x_0)/\theta)^{1/3}$$

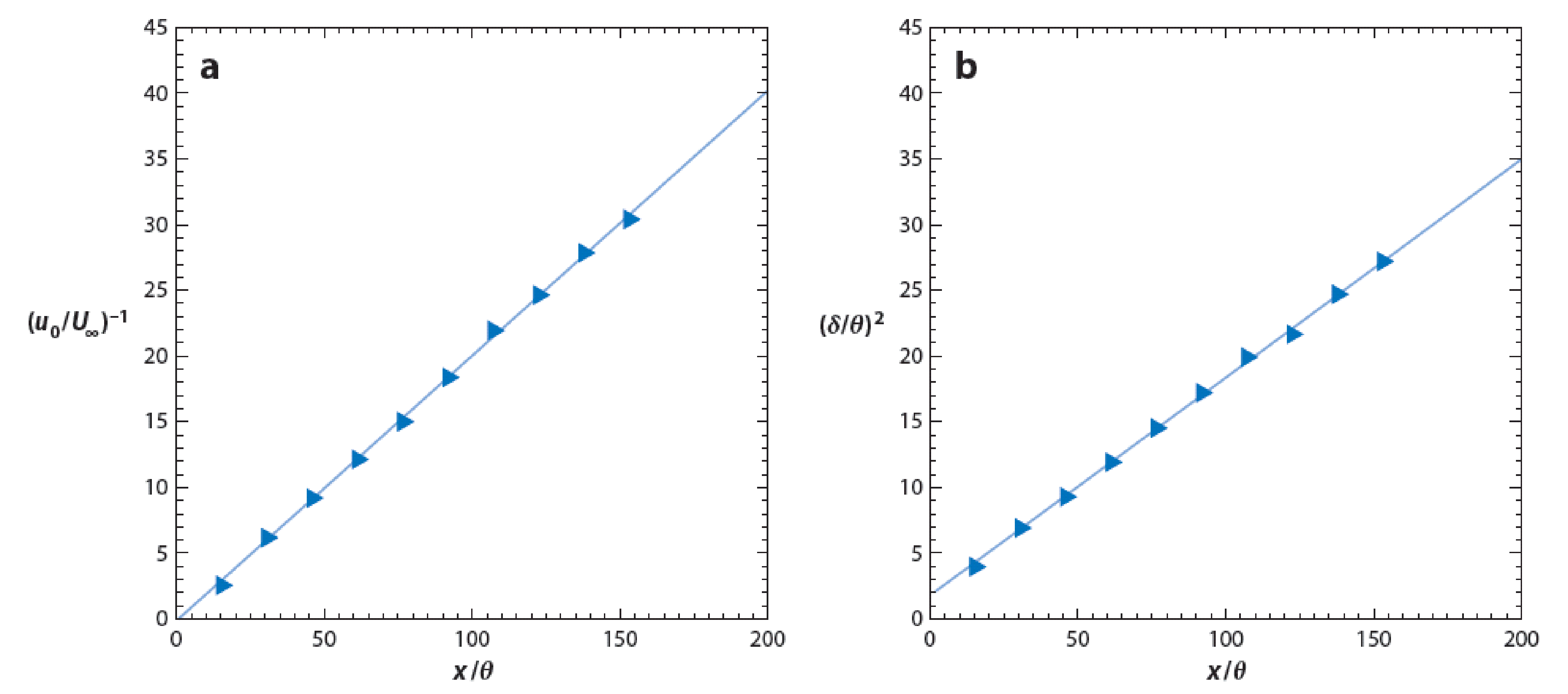
where U_∞ is the upstream velocity and θ is the momentum thickness.

- The new scaling gives:

$$u_0/U_\infty \sim ((x-x_0)/\theta)^{-1} \qquad \delta/\theta \sim ((x-x_0)/\theta)^{1/2}$$

Implications for e.g. wake scaling

Velocity deficit and wake width following the new scalings



Nedic J, Vassilicos JC, Ganapathisubramani B. 2013. Axisymmetric turbulent wakes with new non-equilibrium, similarity scalings. *Phys. Rev. Lett*. 111:144503

The small-scale / topological approach

- The dissipation scaling approach is, at its heart, concerned with the cascade.
- If one resolves turbulence at high resolution, one resolves a set of velocity gradient tensors (VGT). Everything is at small scale, so where is the cascade?

$$\frac{\partial}{\partial t}\mathsf{A} + \mathsf{u}\cdot\nabla\mathsf{A} = -\mathsf{A}^2 - \mathsf{H} + \nu\nabla^2\mathsf{A},$$

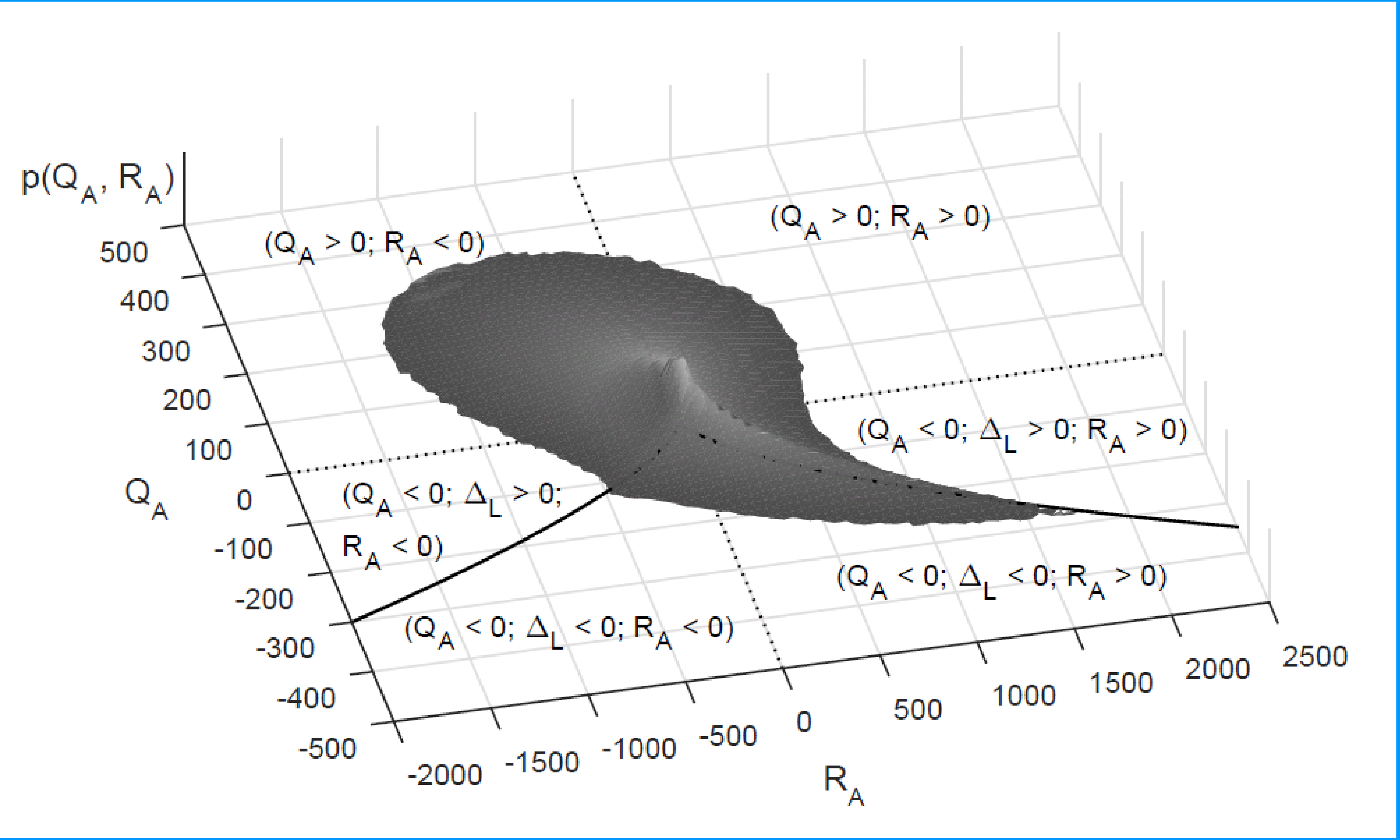
$$\mathsf{A} = \frac{\partial u_i}{\partial x_j} \equiv \begin{pmatrix} \partial u_1/\partial x_1 & \partial u_1/\partial x_2 & \partial u_1/\partial x_3 \\ \partial u_2/\partial x_1 & \partial u_2/\partial x_2 & \partial u_2/\partial x_3 \\ \partial u_3/\partial x_1 & \partial u_3/\partial x_2 & \partial u_3/\partial x_3 \end{pmatrix}$$

$$H_{ij} = \frac{\partial^2 p}{\partial x_i \partial x_j}$$

- The pressure Hessian can be decomposed into an isotropic (local) and deviatoric (non-local) part

$$\mathsf{H} = \frac{Q}{3}\mathsf{I} + \eta$$

Analysis of the VGT in terms of its invariants



The small-scale / topological approach

- The dynamics for the local part with viscosity neglected were considered by Cantwell (1992):

$$\frac{dQ}{dt} = -3R$$

$$\frac{dR}{dt} = \frac{2}{3}Q^2$$

$$Q = \frac{1}{2}(||\Omega||^2 - ||S||^2)$$

$$R = -\det(S) - \mathrm{tr}(\Omega^2 S)$$

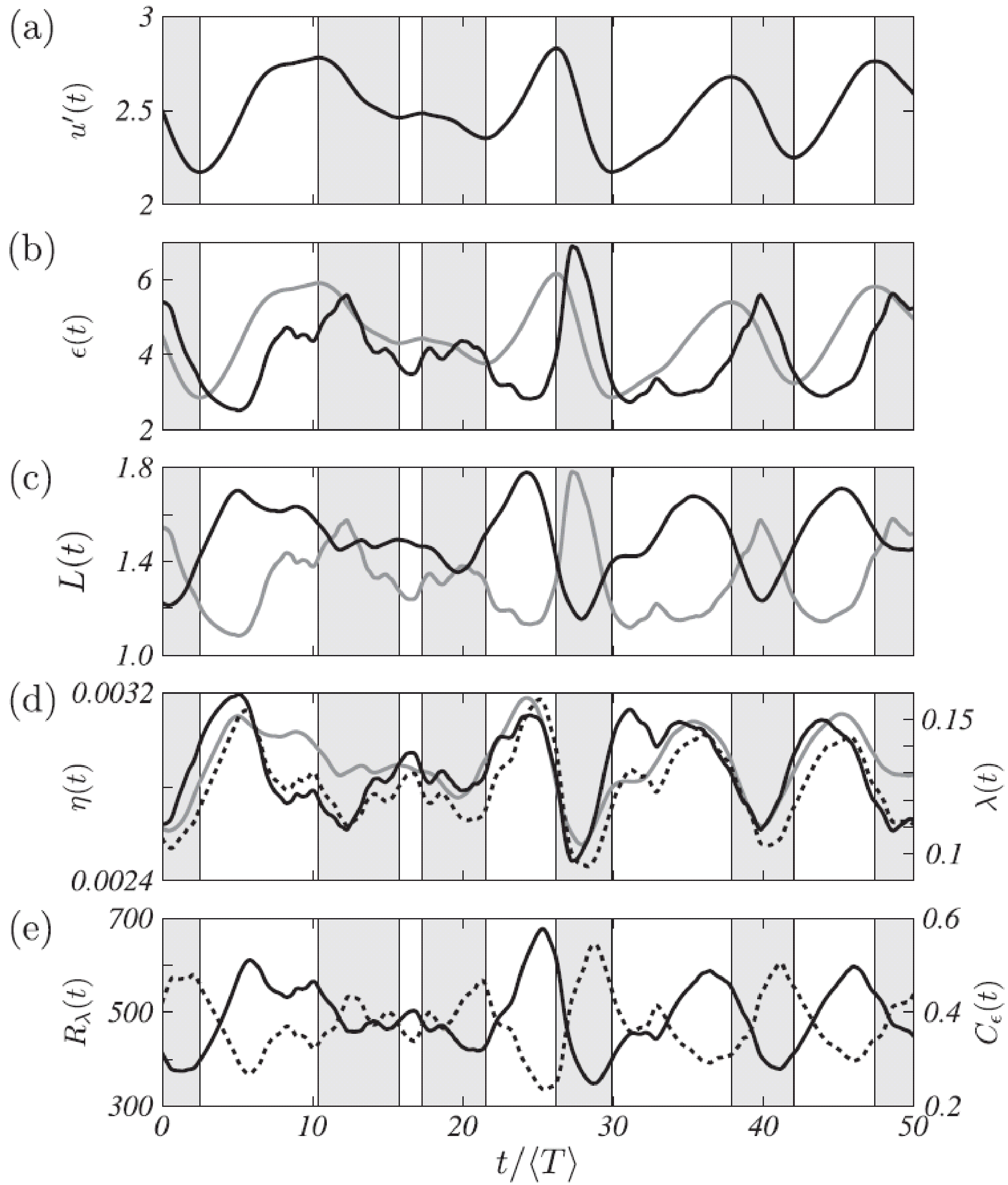
- Open questions then concern the vortical configurations (topology) and non-local effects that result in changes to the dissipation scaling.
- Improvements here will enhance pressure-strain coupling representation in one-point (RANS) closures.

Timescale to establish a cascade

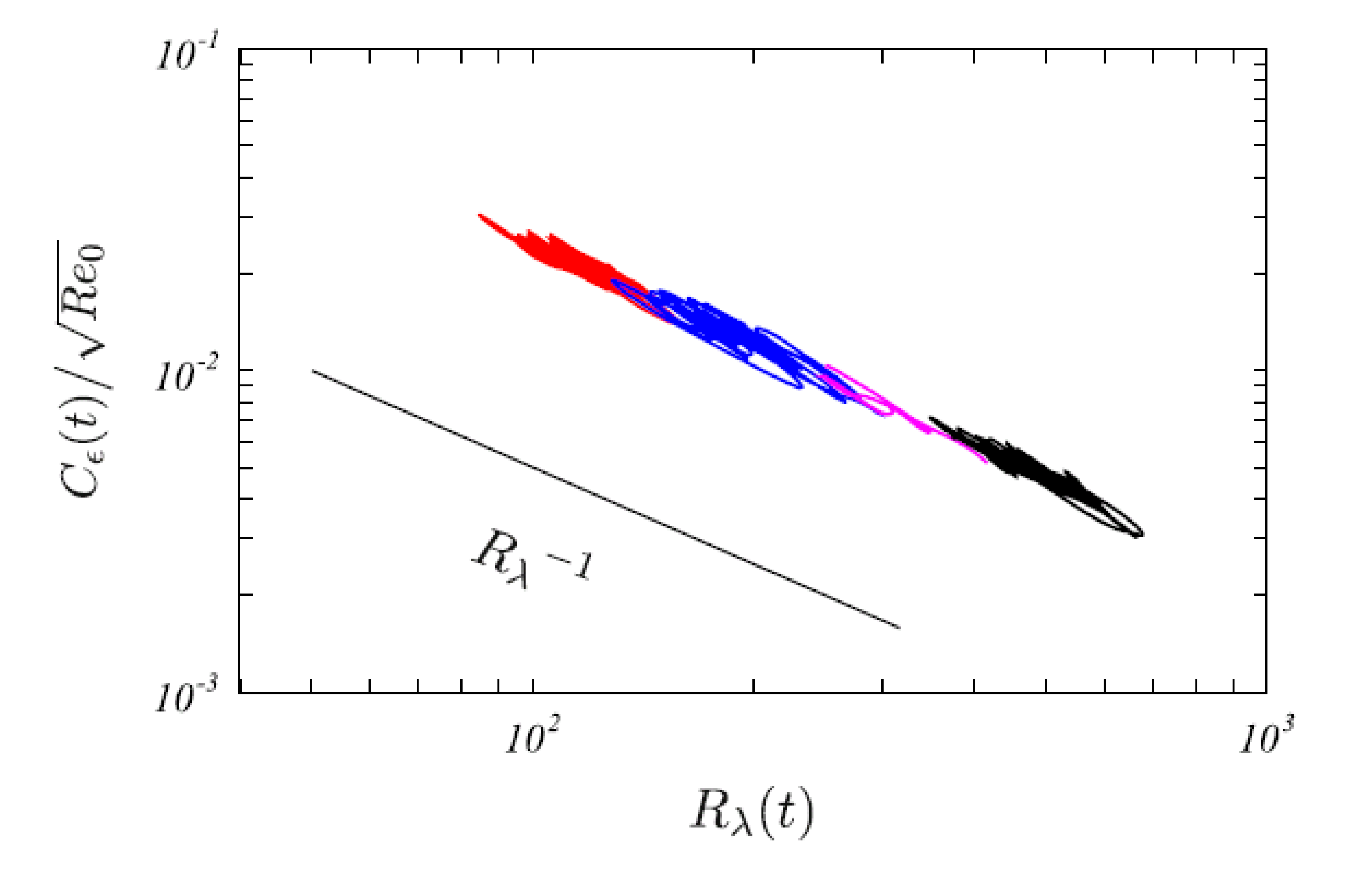
Spatially periodic forcing using DNS:

$$\boldsymbol{f} = f_0 \sin(2\pi y/L_{\mathrm{b}})\, \boldsymbol{e}_x$$

$$\boldsymbol{f}(\boldsymbol{x})(=\boldsymbol{f}(x, y, z))$$



Goto and Vassilicos (2016) *Fluid Dyn. Res.* 48, 2, 021402



Goto and Vassilicos (2016) *Fluid Dyn. Res.* 48, 021402

$$K^{>}(k,\, t) = \int_k^{\infty} E\,(k',\, t)\; \mathrm{d}k'$$

$$\epsilon^{>}(k,\, t) = 2\nu \int_k^{\infty} k'^2 E\,(k',\, t)\; \mathrm{d}k'$$

$$\frac{\partial K^{>}}{\partial t} = \Pi - \epsilon^{>}$$

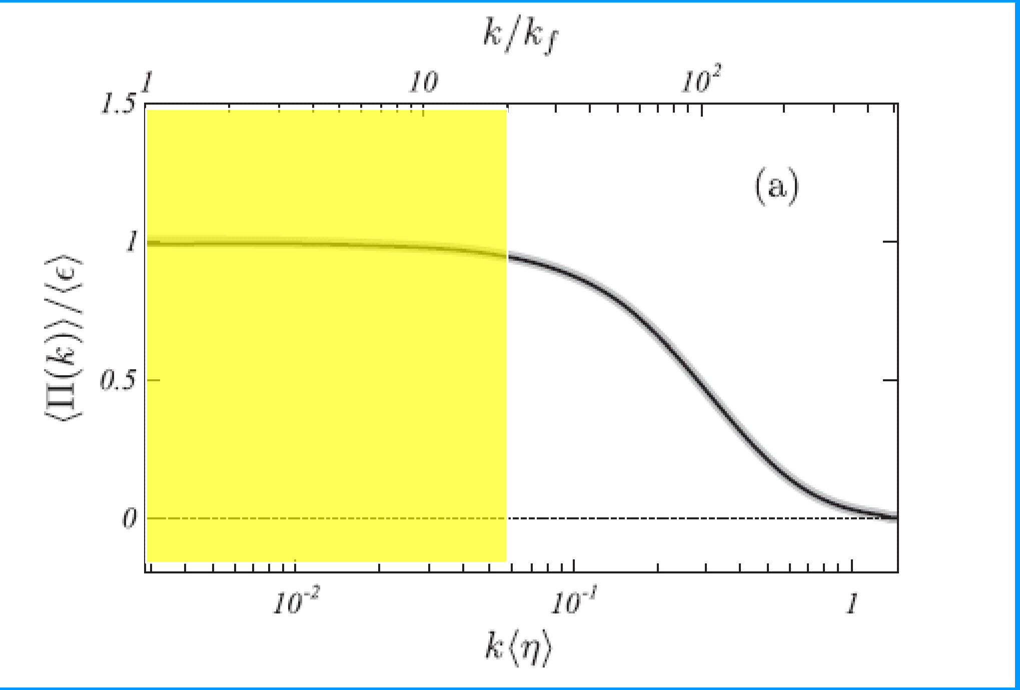
$$\frac{\partial E}{\partial t} = -\frac{\partial \Pi}{\partial k} - 2\nu k^2 E + P,$$

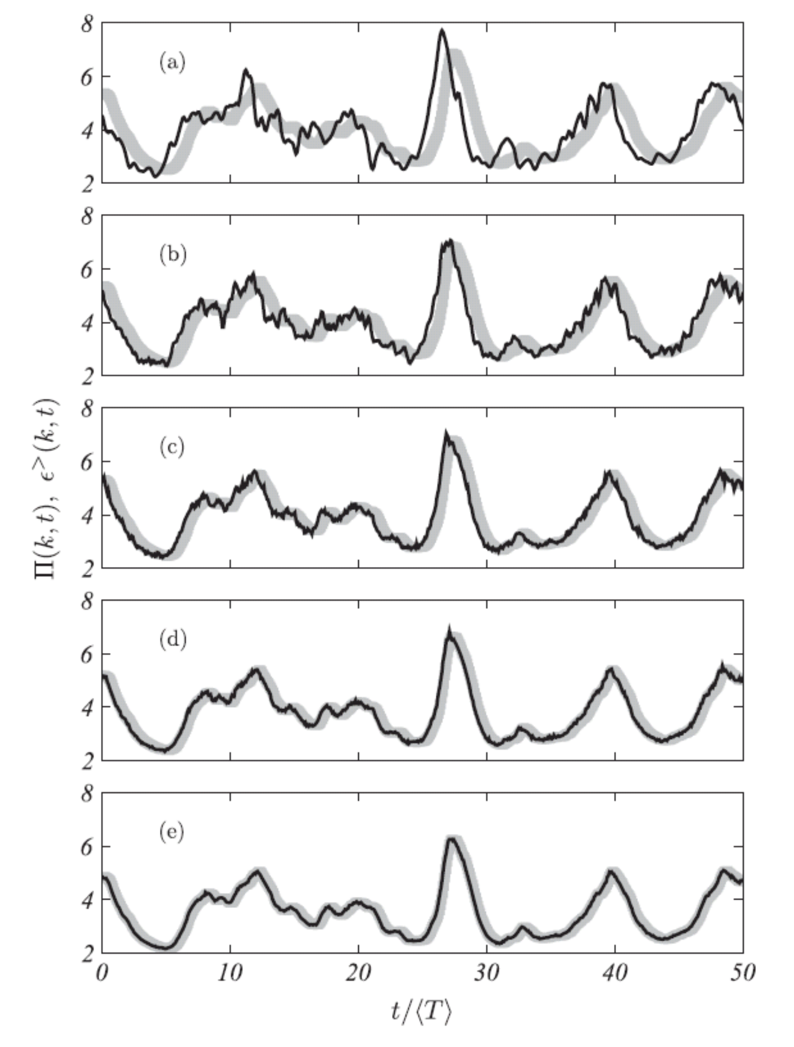
Energy flux function

Power of external force

(Note that equilibrium implies that for slow change in *K*, energy flux and dissipation rate are about equal)

Goto and Vassilicos (2016) *Fluid Dyn. Res.* 48, 021402





Absence of equilibrium seen primarily at low wave numbers (Yoshizawa, 1994, PRE 49, 4065)

Goto and Vassilicos (2016) *Fluid Dyn. Res.* 48, 021402

Coupling to the macro-scale velocity

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K62 introduces potential macroscale dependence not in K41 (K62 is not just dealing with Landau's objection)

I shall show that the equation

$$B_{dd}(r) = C r^{\frac{2}{3}} \bar{\epsilon}^{\frac{2}{3}}$$

from Kolmogorov (1941 *a*) now takes the modified form

$$B_{dd}(r) = C(\mathbf{x}, t)\, r^{\frac{2}{3}} \bar{\epsilon}^{\frac{2}{3}} (L/r)^{-k}, \tag{3}$$

where k is the constant in equation (2) and the factor $C(\mathbf{x}, t)$ depends on the macrostructure of the flow; and that the theorem of constancy of skewness

$$S(r) = B_{ddd}(r)/B_{dd}^{\frac{3}{2}}(r)$$

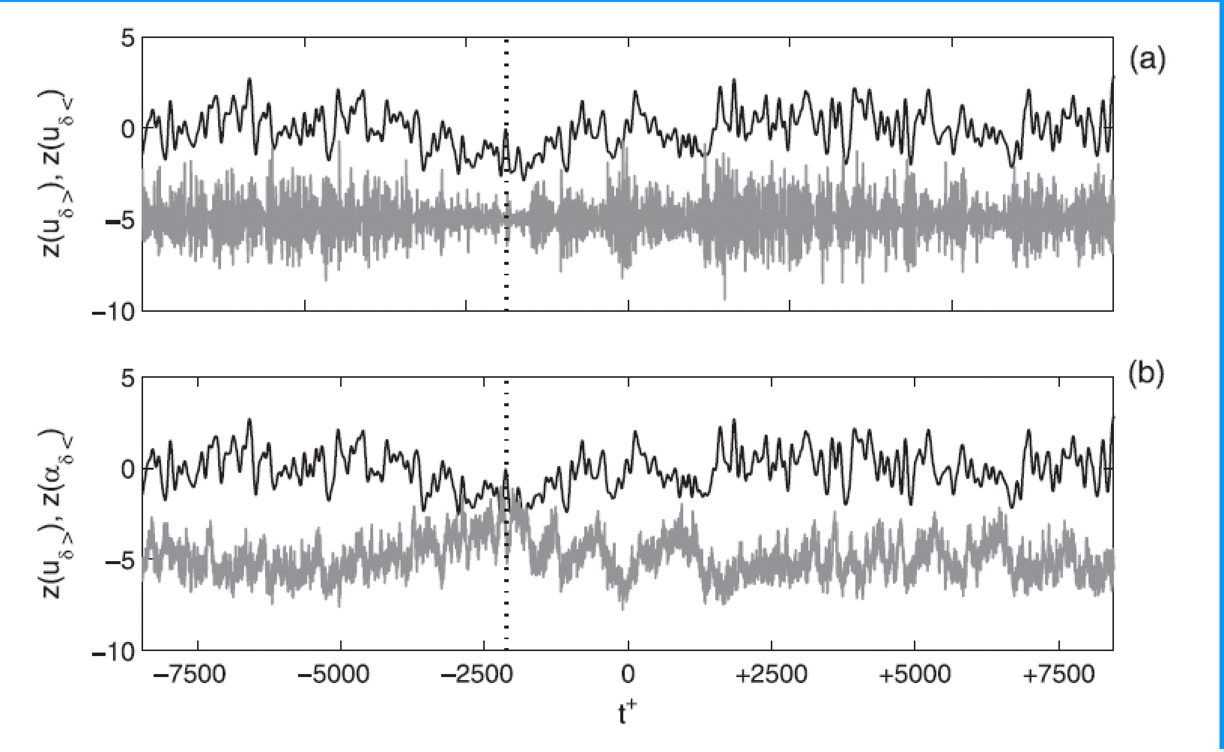
when $l \ll r \ll L$, derived in Kolmogorov (1941 *b*), is now replaced by the equation

$$S(r) = S_0 (L/r)^{\frac{3}{2}k} \tag{4}$$

where the coefficient S_0 also depends on the macrostructure of the flow.

Boundary-Layer Structure

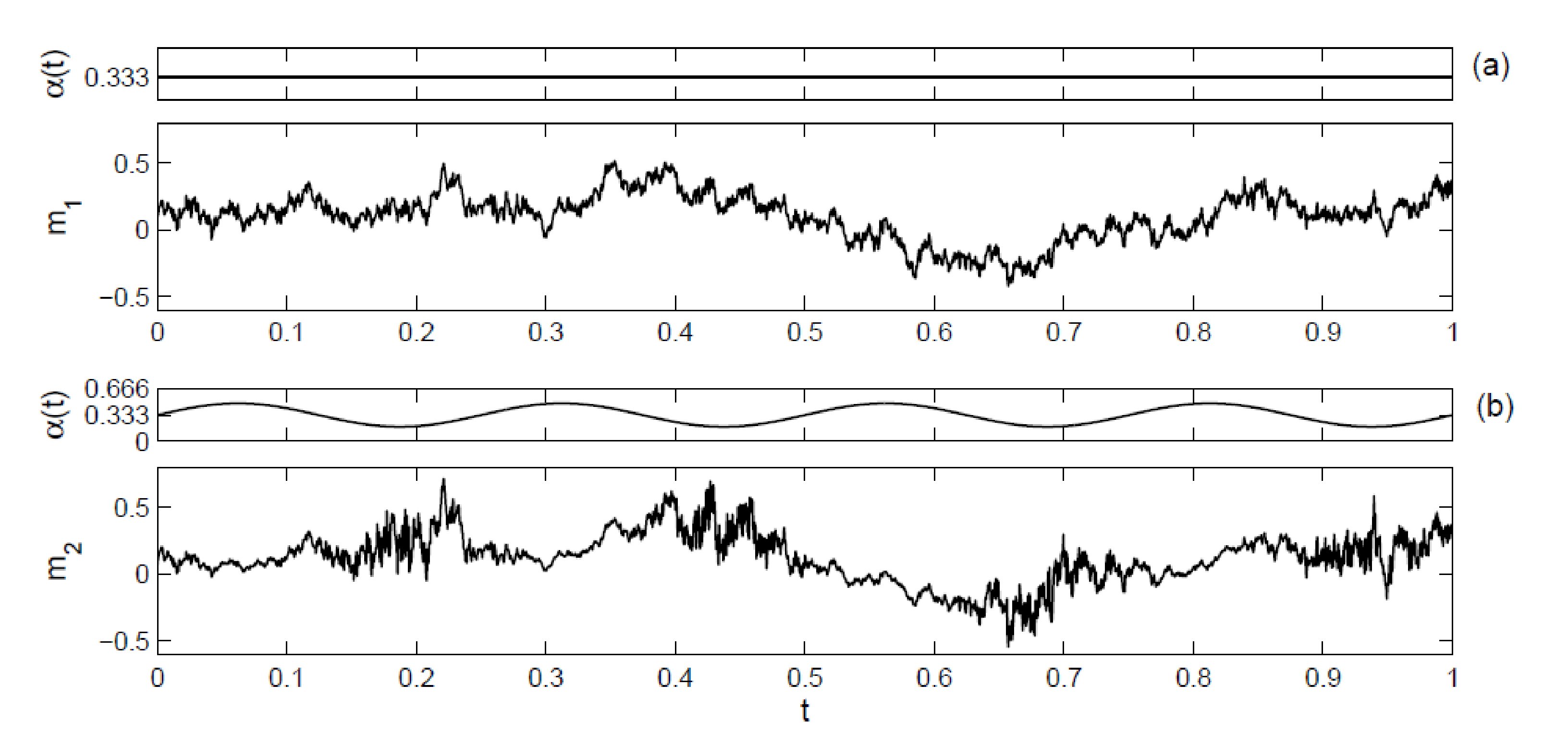
Modulation of the small scales by the large is now a well-known property of boundary-layers, with early work on this due to Townsend (attached eddy hypothesis)



$$t^+ = tU_\tau^2/\nu$$

Keylock, C.J., Ganapathasubramani, B., Monty, J., Hutchins, N. and Marusic, I. 2016. The coupling between inner and outer scales in a zero pressure boundary layer evaluated using a Hölder exponent framework, *Fluid Dynamics Research* 48, 2, 021405.

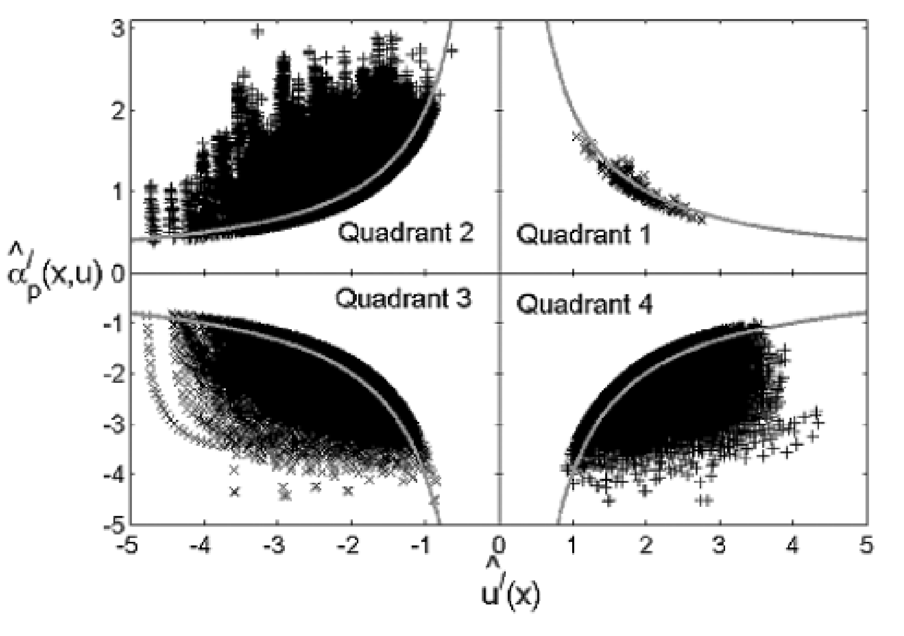
Applied Velocity-Intermittency Analysis



Frisch-Parisi Conjecture:

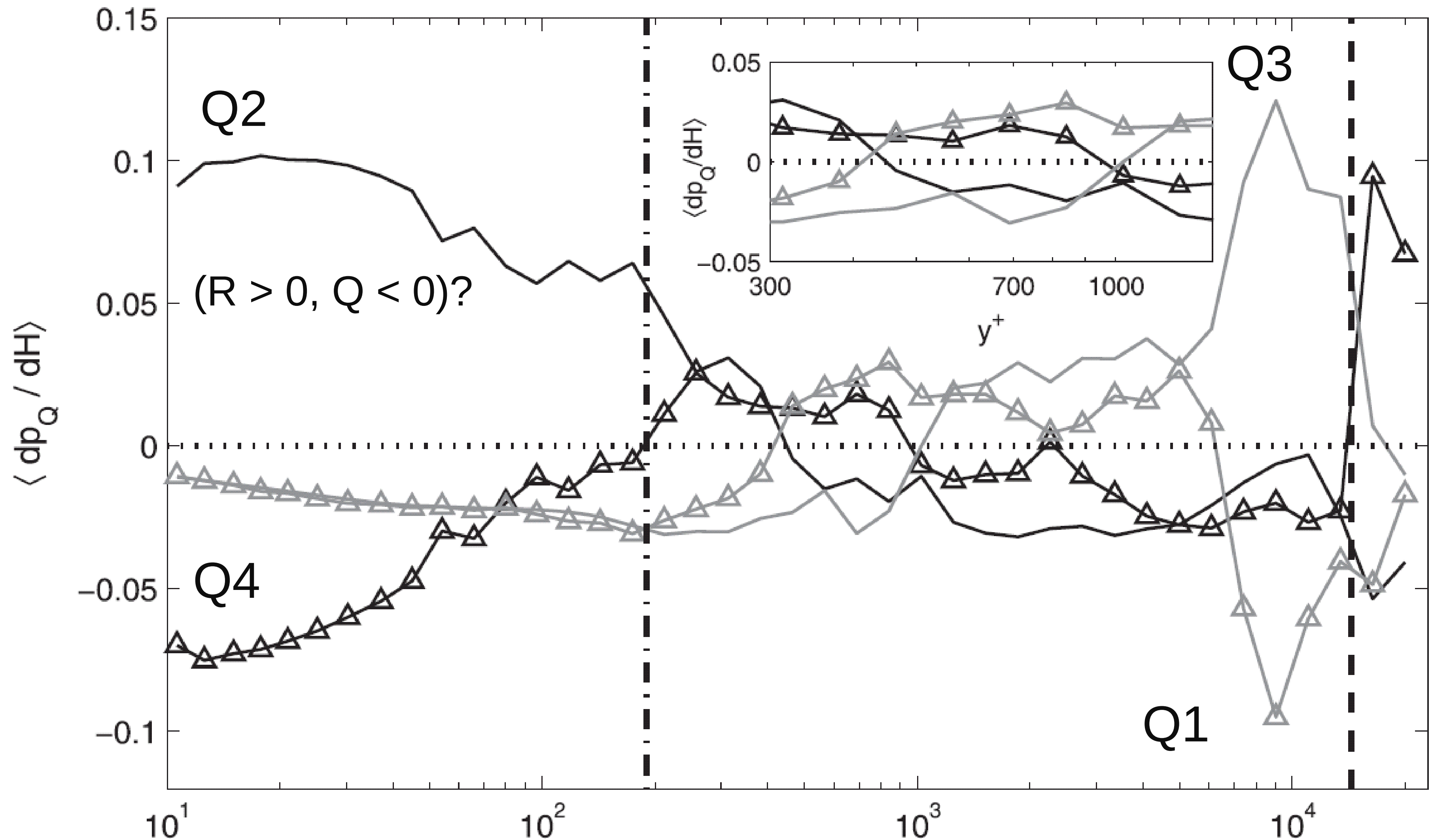
$$D(\alpha_u) = \min_n (\alpha_u n - \xi_n + 1)$$

Velocity-Intermittency Analysis



Quadrant number (Q)	$sgn(u'_{\delta>})$	$sgn(\alpha'_{\delta<})$
1	+	+
2	−	+
3	−	−
4	+	−

Keylock, C.J., Nishimura, K., Peinke, J. 2012. *Journal of Geophysical Research* 117, F01037, doi:10.1029/2011JF002127.



$$y^+ = yU_t/\nu$$

DNS by Naso et al. (2006) suggests the Vieillefosse tail grows preferentially at high shear rates consistent with this.

Conclusion

- An energy cascade with a -5/3 slope is seen in many flows that seem to violate the assumptions of the Richardson-Kolmogorov cascade. Why?
- New dissipation scalings have been observed in experiments and DNS and suggest new means to write down closures.
- Wave turbulence with its large-scale, (pseudo)-periodic forcing is perhaps in a state where a cascade is continually being re-established as the forcing waxes and wanes.
- Non-equilibrium effects are therefore worthy of further investigation.

