Statistical and spectral properties of the modulation instability: experiments and modelling by using soliton gas

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Guillaume Michel, Gaurav Prabhudesai (ENS, PhD), *Ecole Norm. Sup., France*
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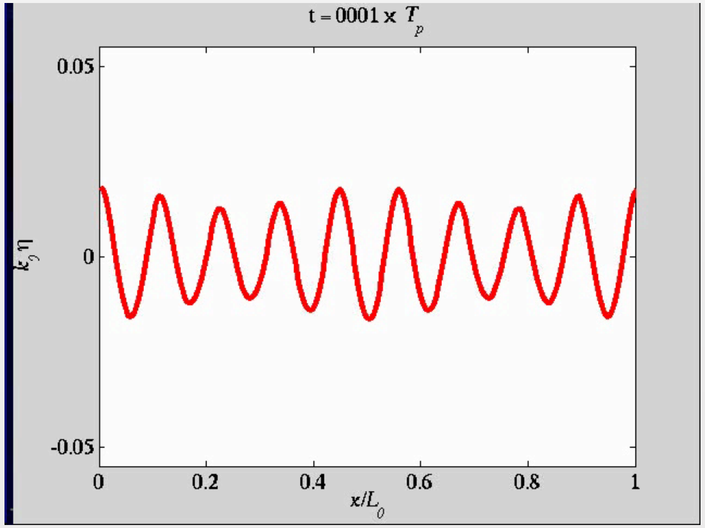
Amin Chabchoub, *Univ. Of Sydney, Australia*

WCST2019, Norwich (UK), 30 Oct-1st Nov. 2019

Modulation Instability

- Benjamin-Feir instability (1967)
- Deep Water waves
- Sideband instability (breathers)

N. Akhmediev *et al.*,, Sov. Phys. JETP 62, 894 (1985).
N. Akhmediev and V. Korneev,, Theor. Math. Phys. 69, 1089 (1986).
N. Akhmediev, *et al.* Phys. Lett. A 373, 675 (2009).



from Nobuhito Mori, reuse with permission

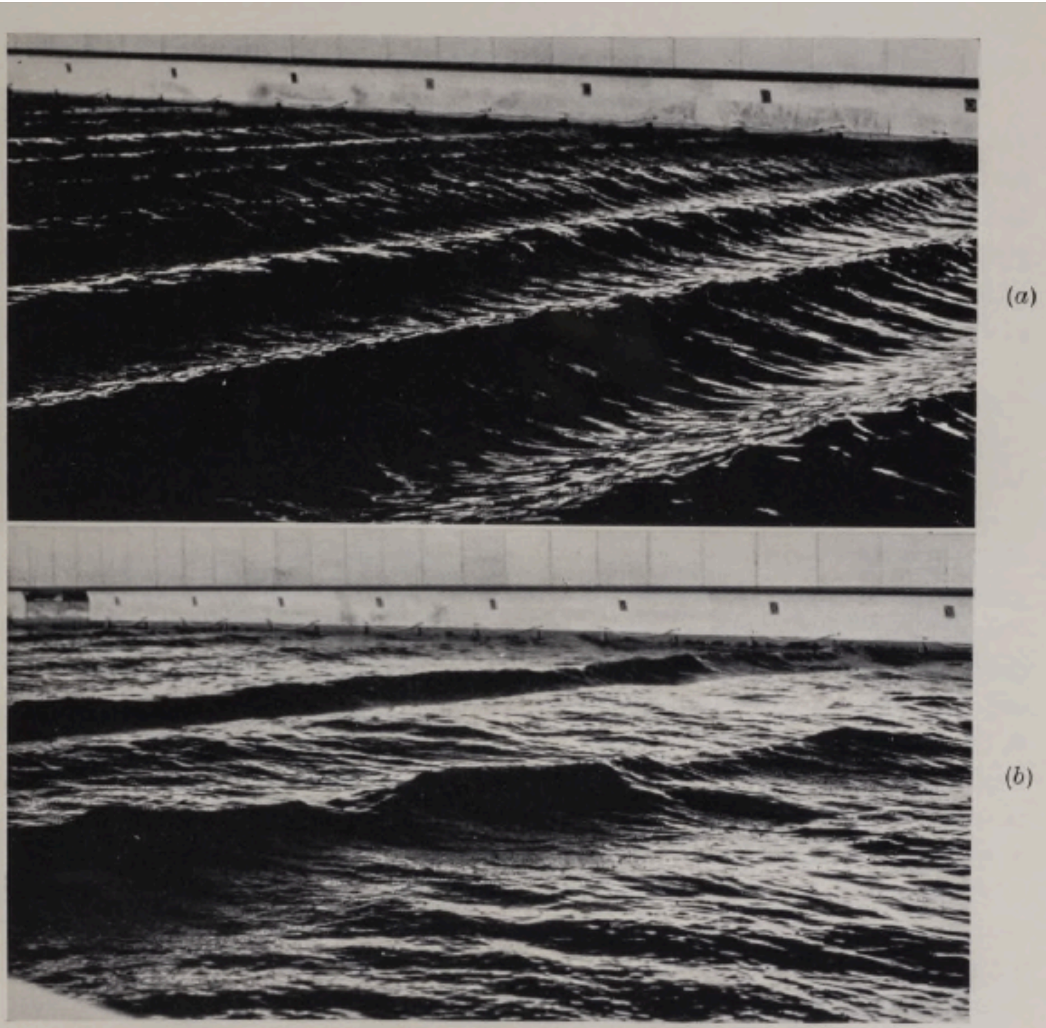
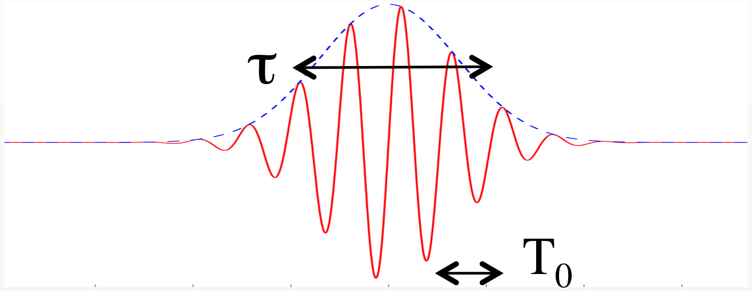


FIGURE 1. Photographs of a progressive wavetrain at two stations, illustrating disintegration due to instability: (*a*) view near to wavemaker; (*b*) view at 200 ft. farther from wavemaker. Fundamental wavelength, 7·2 ft.

Benjamin, T. Brooke; Feir, J.E. (1967). Journal of Fluid Mechanics. 27 (3) p.417–430
Benjamin, T.B. (1967). Proceedings of the Royal Society of London. A. 299 (1456) p.59–76

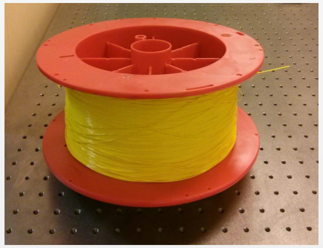
Focusing 1D nonlinear Schrodinger equation

$$\frac{\partial\psi}{\partial z} = \frac{1}{2}\frac{\partial^2\psi}{\partial t^2} + i|\psi|^2\psi$$



- **Nonlinear optics (e.g. fibers)**

$$E(x,y,z,t) = \Re\left(A(x,y)\,\boxed{\psi(z,t)}\,e^{i(k_0 z-\omega_0 t)}\right)$$

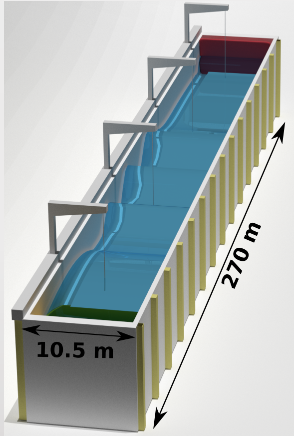


T0 ~ 5 fs
τ ~ ps
L ~ 0.1-1 km

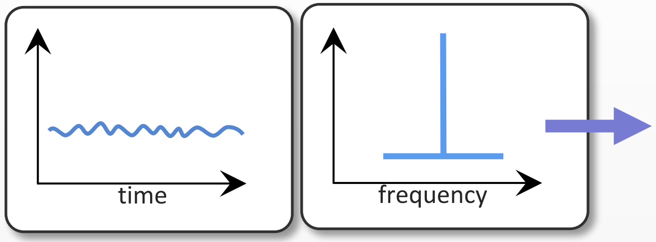
- **Deep water waves**

$$\eta(z,t) = \Re\left(\boxed{\psi(z,t)}\,e^{i(k_0 z-\omega_0 t)}\right)$$

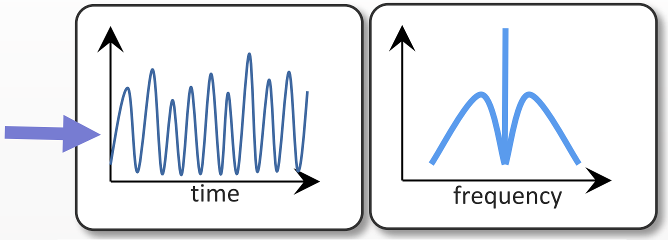
T0 ~ s
τ ~ 5s
L ~ 0.1 km



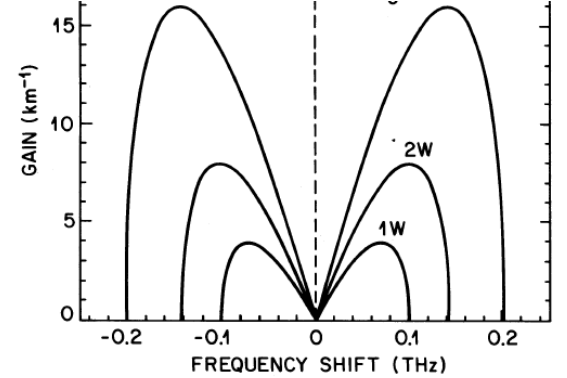
Spontaneous (noise induced) Modulation Instability



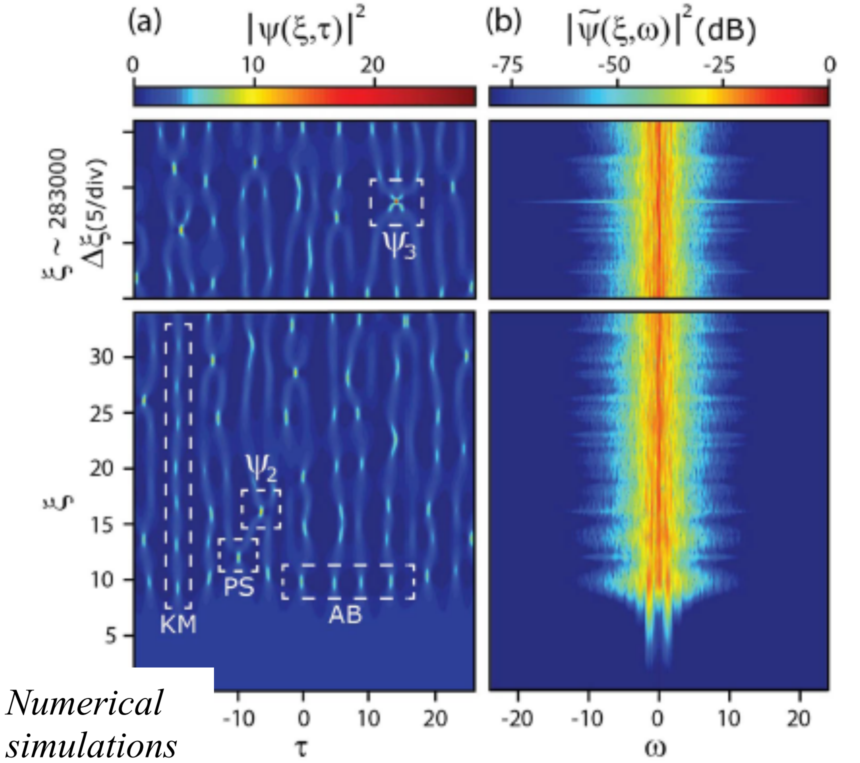
$$\frac{\partial \psi}{\partial z} = \frac{1}{2}\frac{\partial^2 \psi}{\partial t^2} + i|\psi|^2\psi$$



$$g(\omega) = |\omega|\sqrt{4-\omega^2}$$

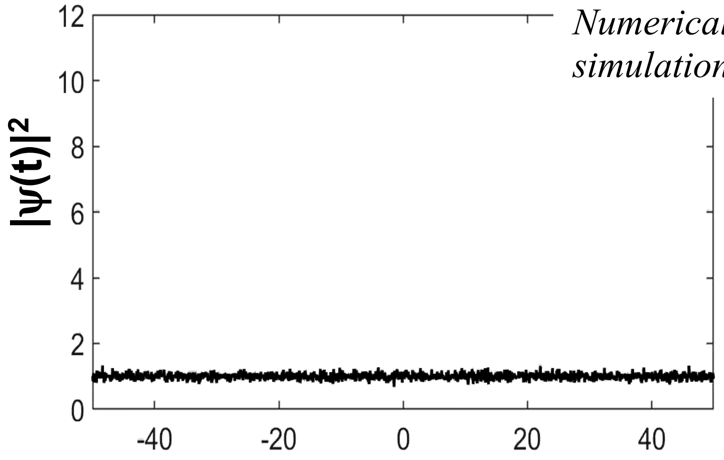


- Local emergence of breathers



Numerical simulations

Toenger, S., *et al.* J. M. Scientific reports, 5, 10380 (2015)

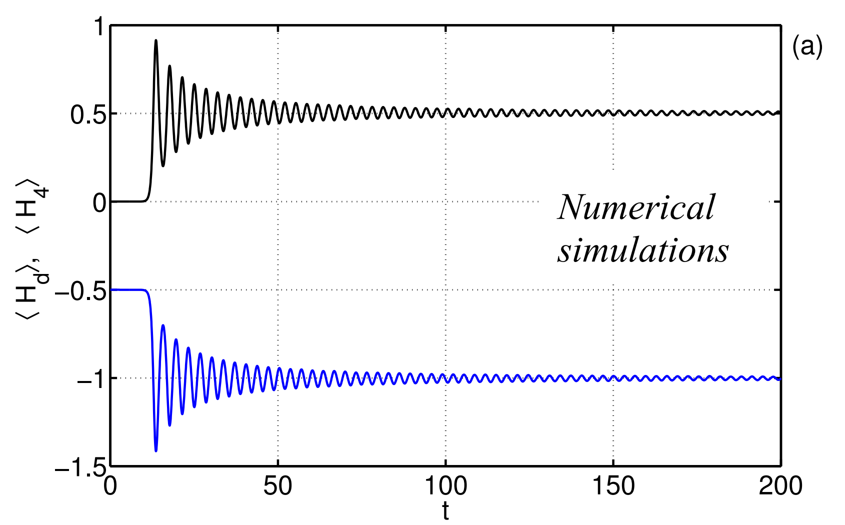


Numerical simulations

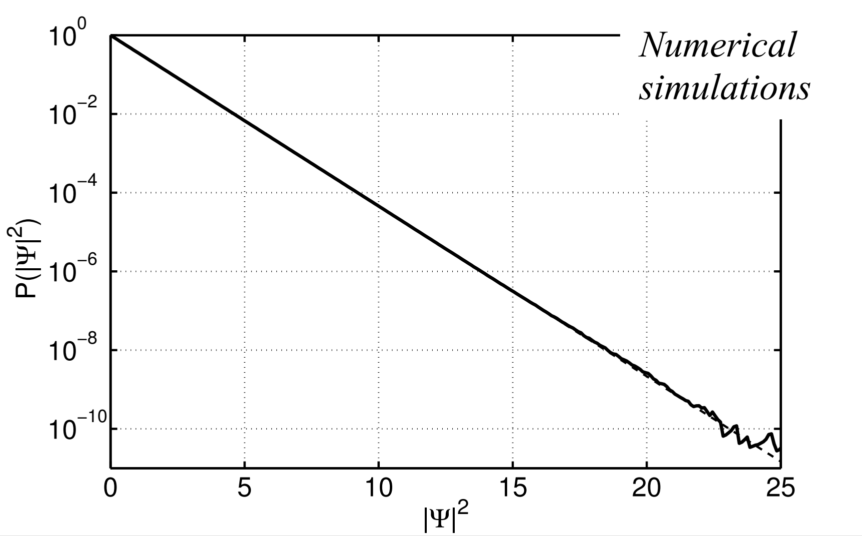
Spontaneous modulation instability: statistics

Agafontsev, D. S., & Zakharov, V. E. *Integrable turbulence and formation of rogue waves,* Nonlinearity, **28**,(8), 2791. (2015)

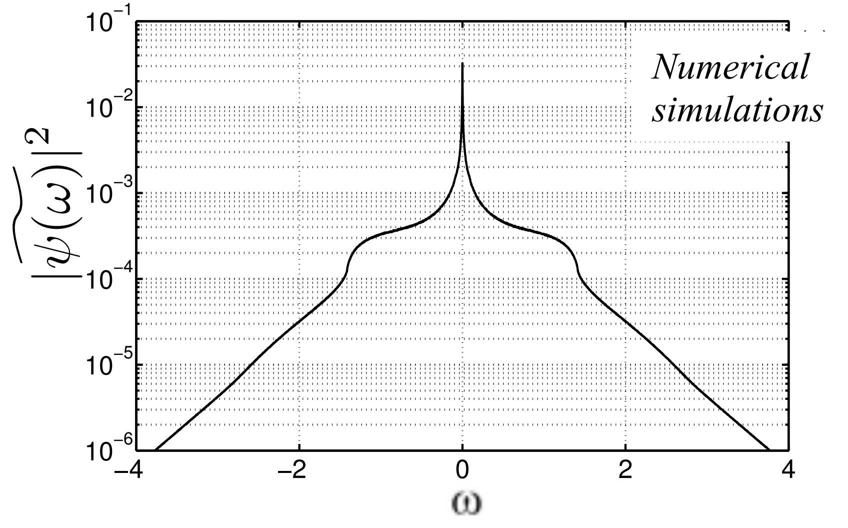
- **Transient regime: oscillations**



- **Long-term statistics : normal law !**

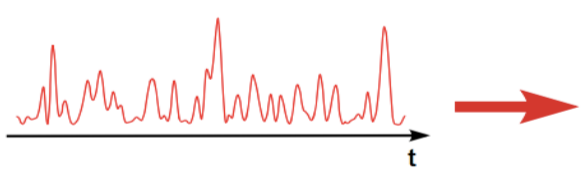


- Stationary spectrum



Integrable Turbulence

- ✓ **Random initial conditions + integrable system (1D-NLSE)**



$$i\frac{\partial\psi}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2\psi}{\partial t^2} - \gamma|\psi|^2\psi$$

NLS, KdV, Sine-Gordon

- ✓ Universal equations
- ✓ Inverse Scattering Transform (IST)
- ✓ Solitons, breathers…

> *"Nonlinear wave systems integrable by Inverse Scattering Method could demonstrate a complex behavior that demands the statistical description. The theory of this description composes a new chapter in the theory of wave turbulence -Turbulence in Integrable Systems"*
>
> *Turbulence in Integrable Systems*, V.E. Zakharov, Studies in Applied Mathematics, **122**, 219 (2009)

D.S. Agafontsev and V.E. Zakharov, Nonlinearity, (2015)

J. Soto-Crespo et al., Phys. Rev. Lett., 2016

P. Walczak et al., Phys. Rev. Lett., **114**, 143903, (2015)
S. Randoux et al, Physica D : Nonlinear Phenomena, **333**, (2016)
P. Suret *et al*. Nat. Commun. **7,** 13136 (2016).
A. Tikan, *et al*., Nat. Photon., **12**, 228 (2018)

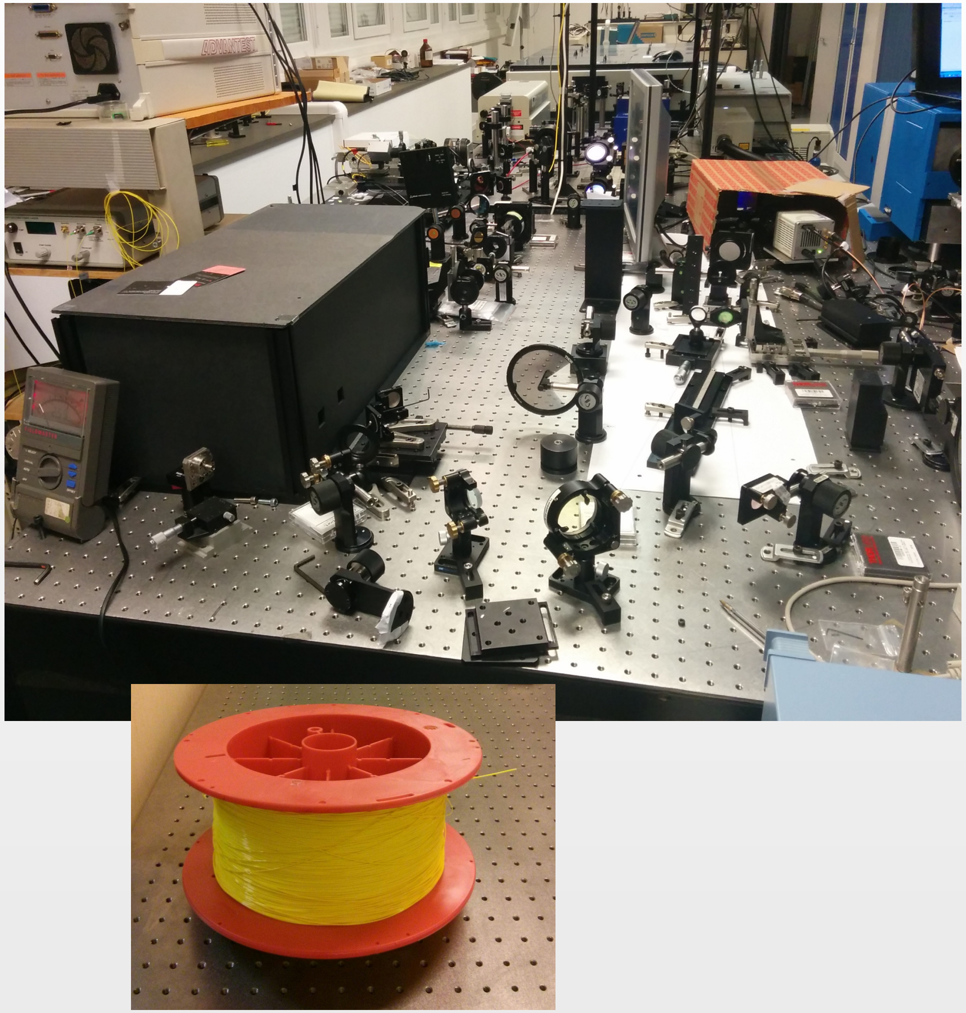
No Resonances… but stationary state

$$k(\omega) = \beta_2\omega^2$$

$$\omega_1 + \omega_2 = \omega_3 + \omega_4$$

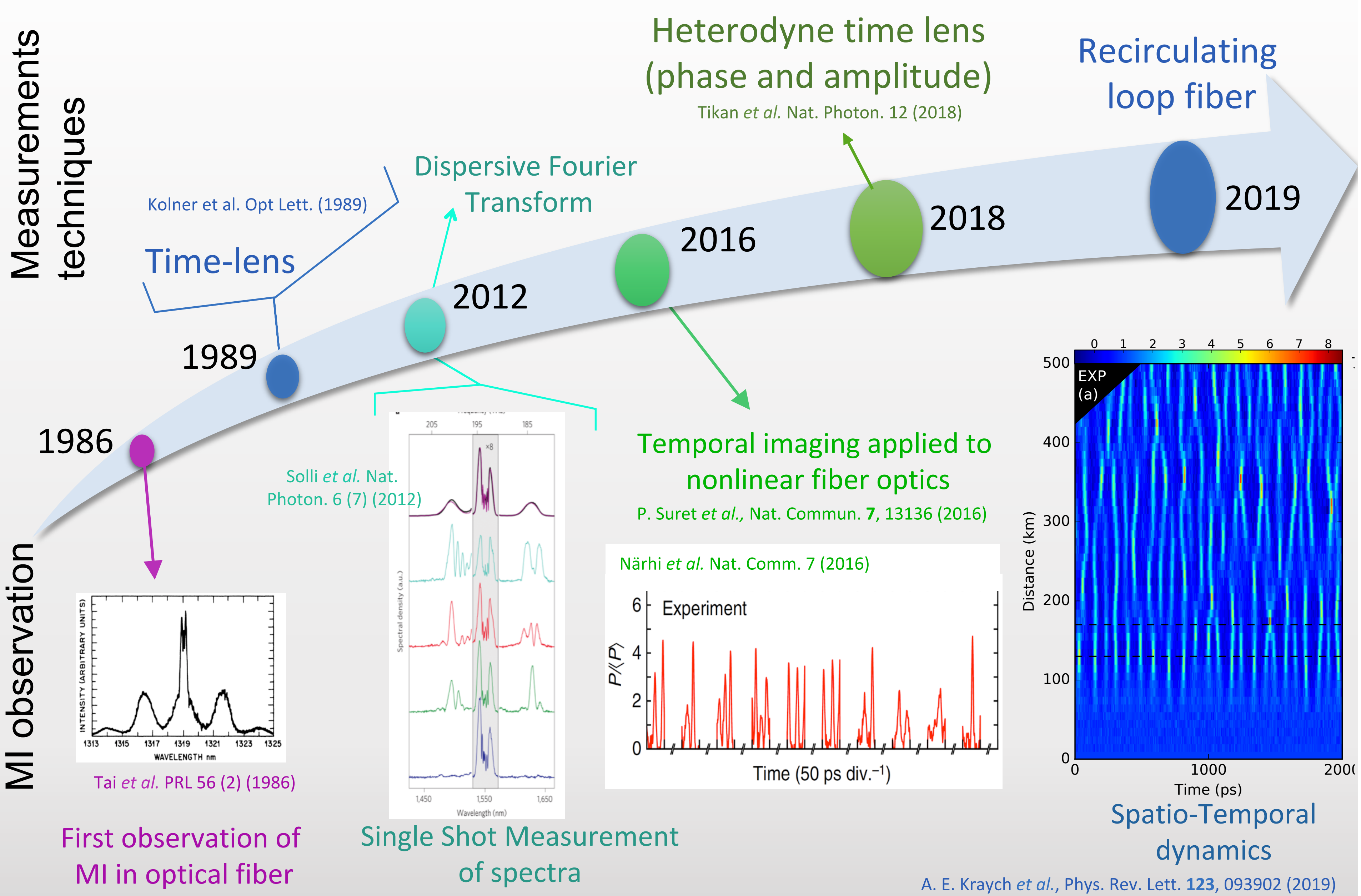
$$k_1 + k_2 \neq k_3 + k_4$$

Experiments in optical fibers and in water tank



- Phlam, University of Lille, France
- Ecole Centrale of Nantes, France

Modulation Instability in optical fiber experiments



Measurements techniques

MI observation

1986

First observation of MI in optical fiber

Tai *et al.* PRL 56 (2) (1986)

1989

Time-lens

Kolner et al. Opt Lett. (1989)

2012

Dispersive Fourier Transform

Single Shot Measurement of spectra

Solli *et al.* Nat. Photon. 6 (7) (2012)

2016

Temporal imaging applied to nonlinear fiber optics

P. Suret *et al.*, Nat. Commun. **7**, 13136 (2016)

Närhi *et al.* Nat. Comm. 7 (2016)

2018

Heterodyne time lens (phase and amplitude)

Tikan *et al.* Nat. Photon. 12 (2018)

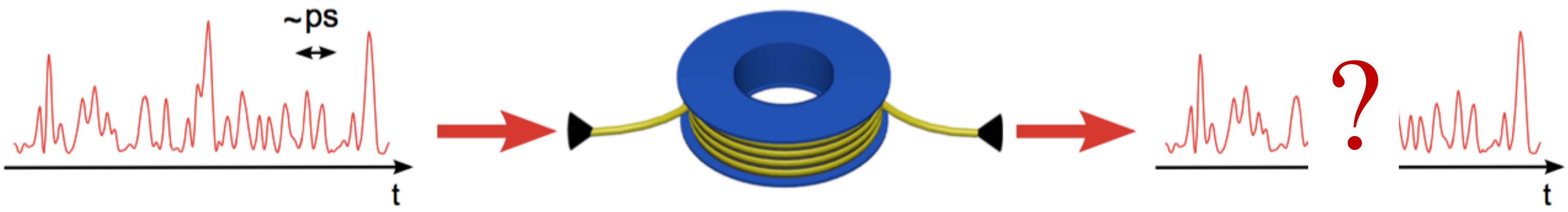
2019

Recirculating loop fiber

Spatio-Temporal dynamics

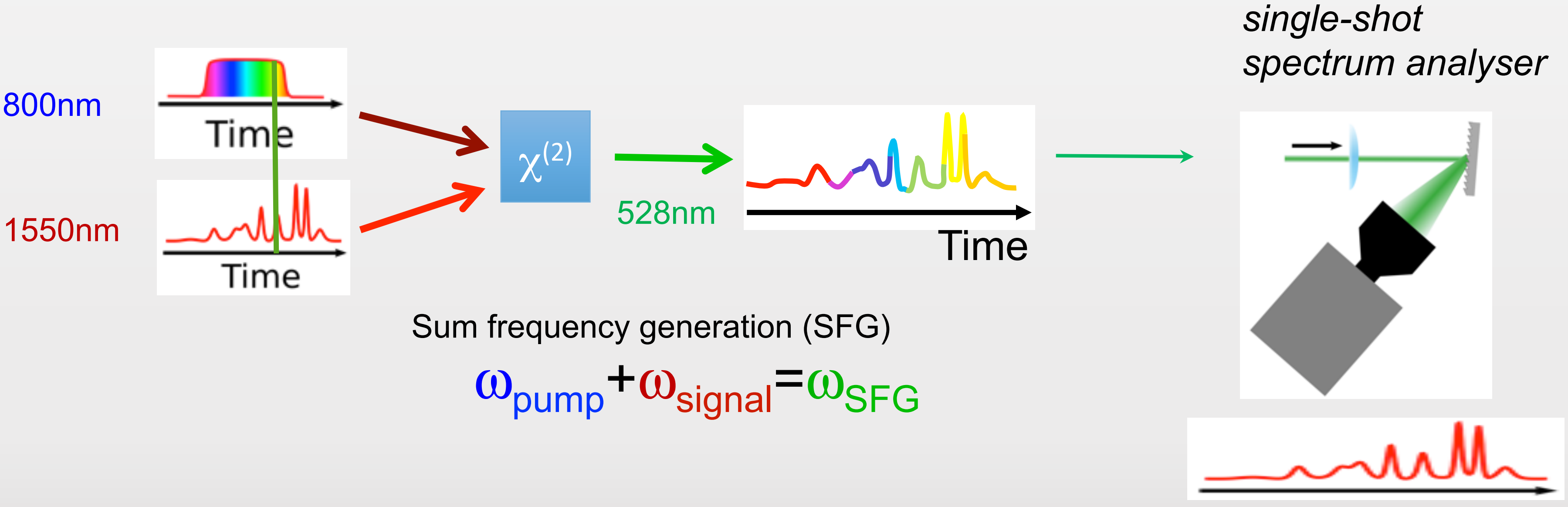
A. E. Kraych *et al.*, Phys. Rev. Lett. **123**, 093902 (2019)

Ultrafast measurement in optical fiber experiments



- ✓ P=10mW / Time scale ~100ps : fast photodetectors and Oscilloscope

- ✓ P=1W / Time scale ~1ps
- ✓ Temporal imaging (SEAHORSE)
- ✓ + Phase



single-shot
spectrum analyser

Sum frequency generation (SFG)

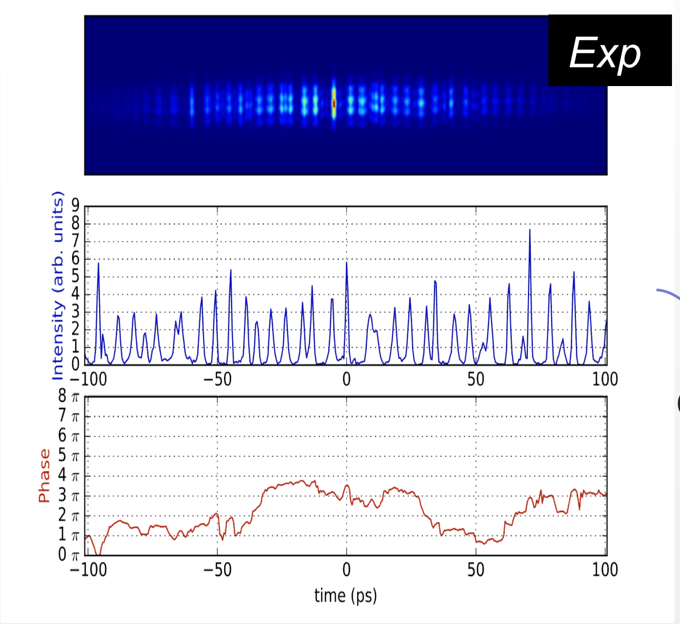
$$\omega_{pump}+\omega_{signal}=\omega_{SFG}$$

space !

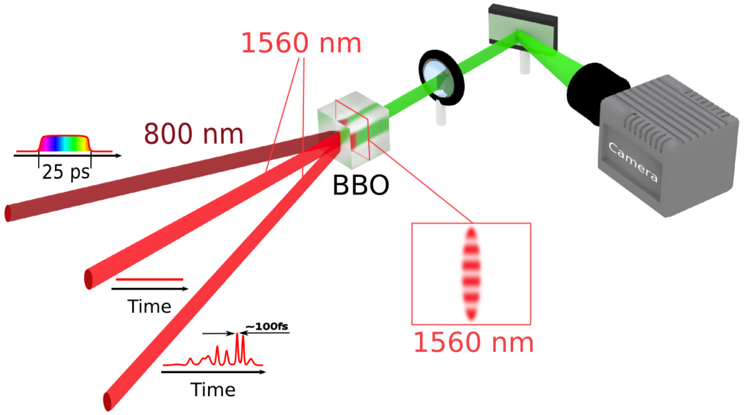
P. Suret *et al.,* Nat. Commun. **7**, 13136 (2016)
A. Tikan *et al.,* Nat. Photon. **12** (2018)

Spontaneous modulation instability in optical fiber experiments

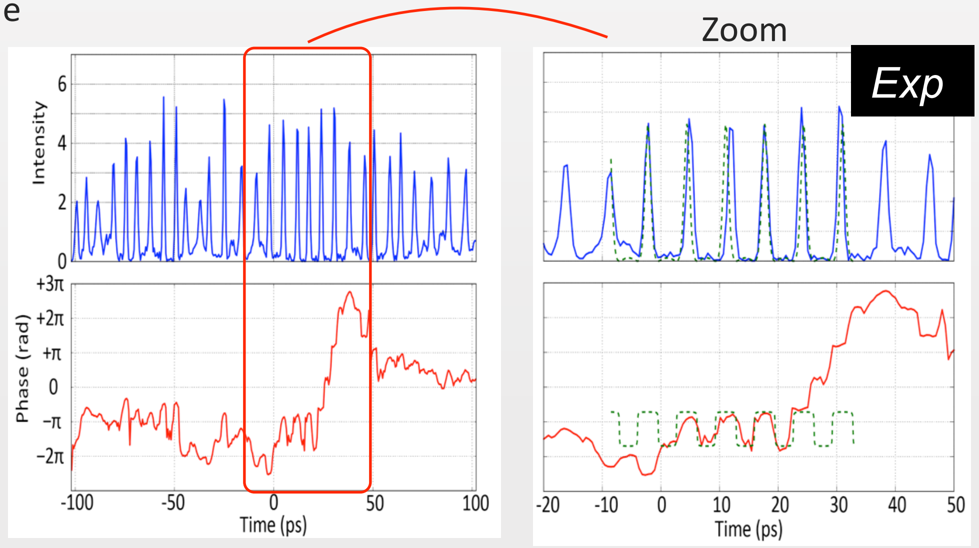
- **Phase and Amplitude ultrafast measurement**

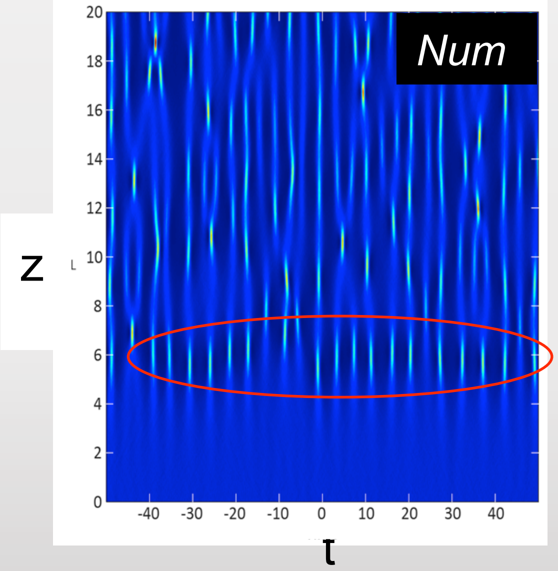


One frame



P. Suret *et al.*, Nat. Commun. **7**, 13136 (2016)
A. Tikan *et al.*, Nat. Photon. **12** (2018)



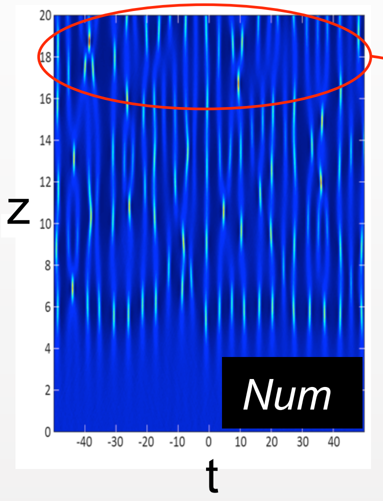


Observations of quasi-periodic structures close to Akhmediev's Breather solution

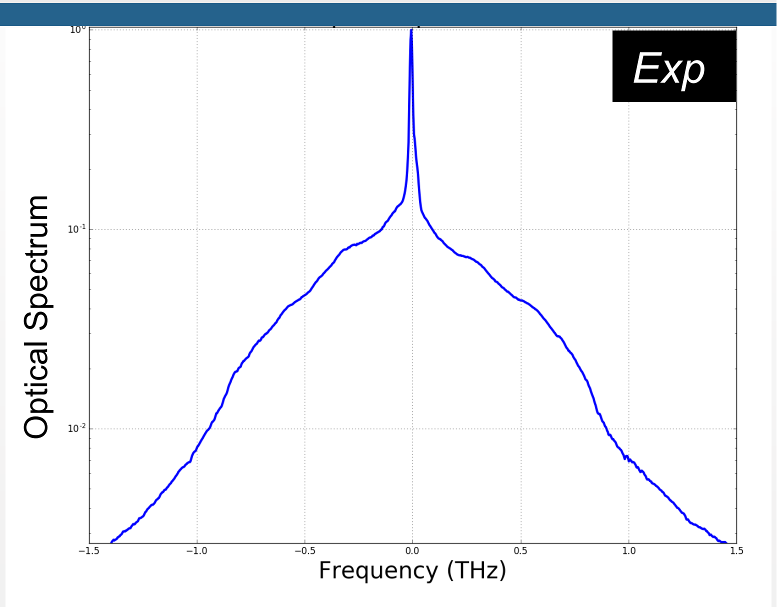
- 500m of SMF-28 (γ = 1,3/W/km, β_2 = -21.7 ps/km^2), P ≈7W , L ≈ 5 x L_{NL}

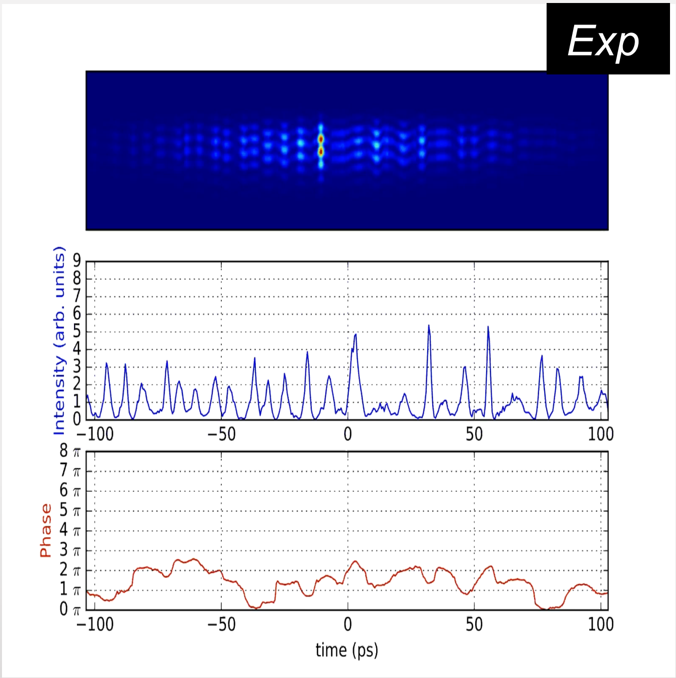
Long term evolution of spontaneous modulation instability (optical fiber experiments)

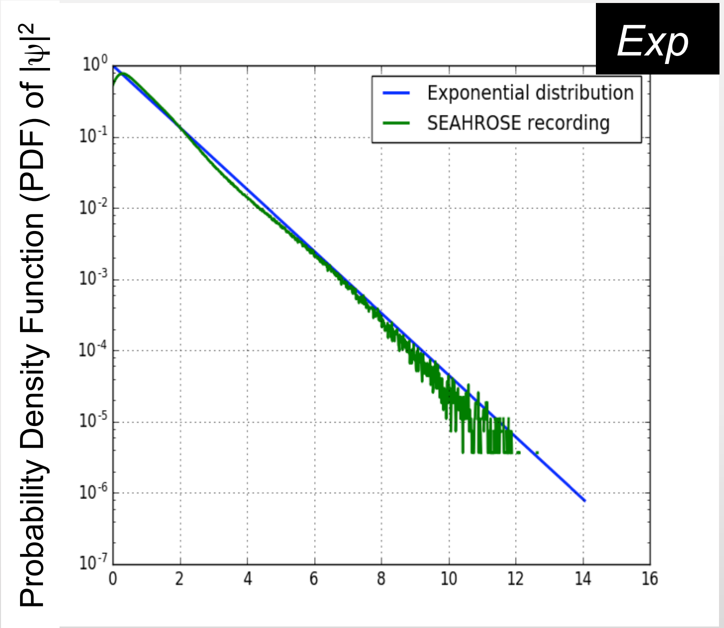
New experiments from A. Lebel et al.
Note, see also : Närhi *et al.* Nat. Comm. 7 (2016)



Stationary (statistical) state





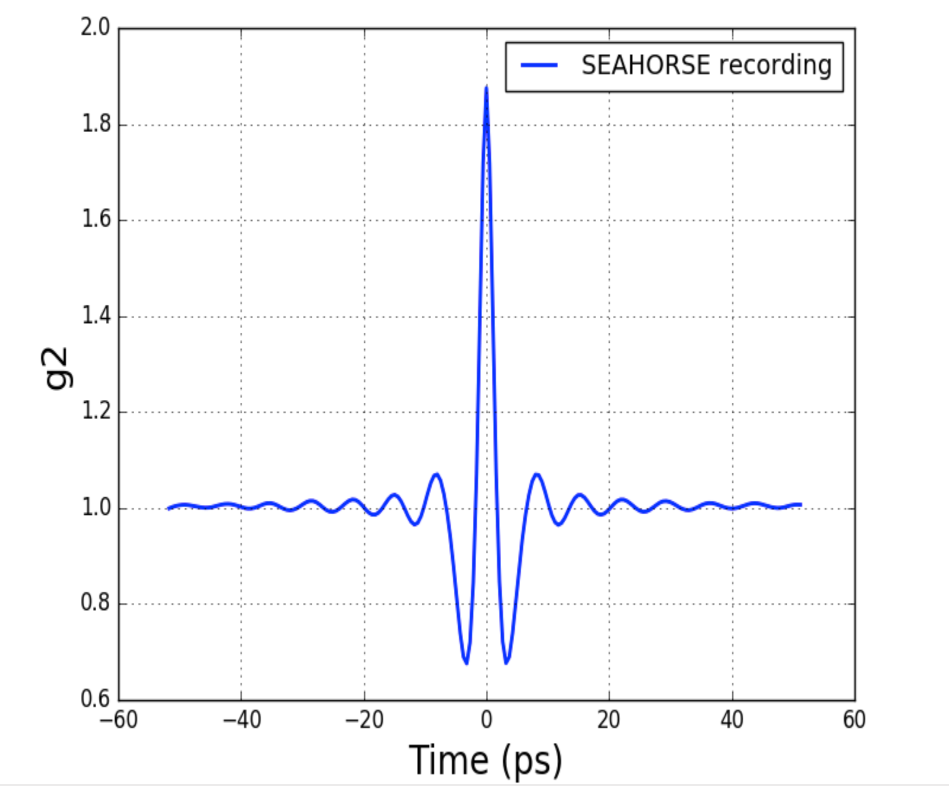


Long term evolution of spontaneous modulation instability (optical fiber experiments)

- **Autocorrelation of power (second order coherence)**

$$g_z^{(2)}(\tau) = \frac{\langle P(z,t)P(z,t+\tau)\rangle}{\langle P(z,t)\rangle^2}$$

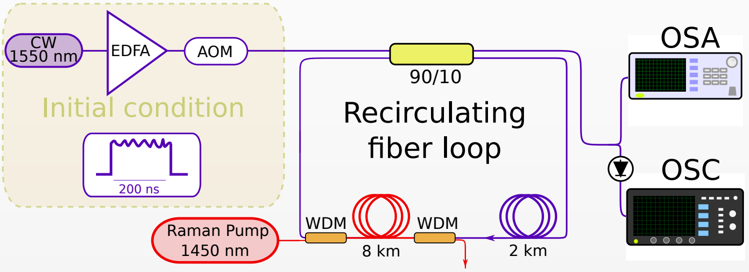
$$P = |\psi|^2$$

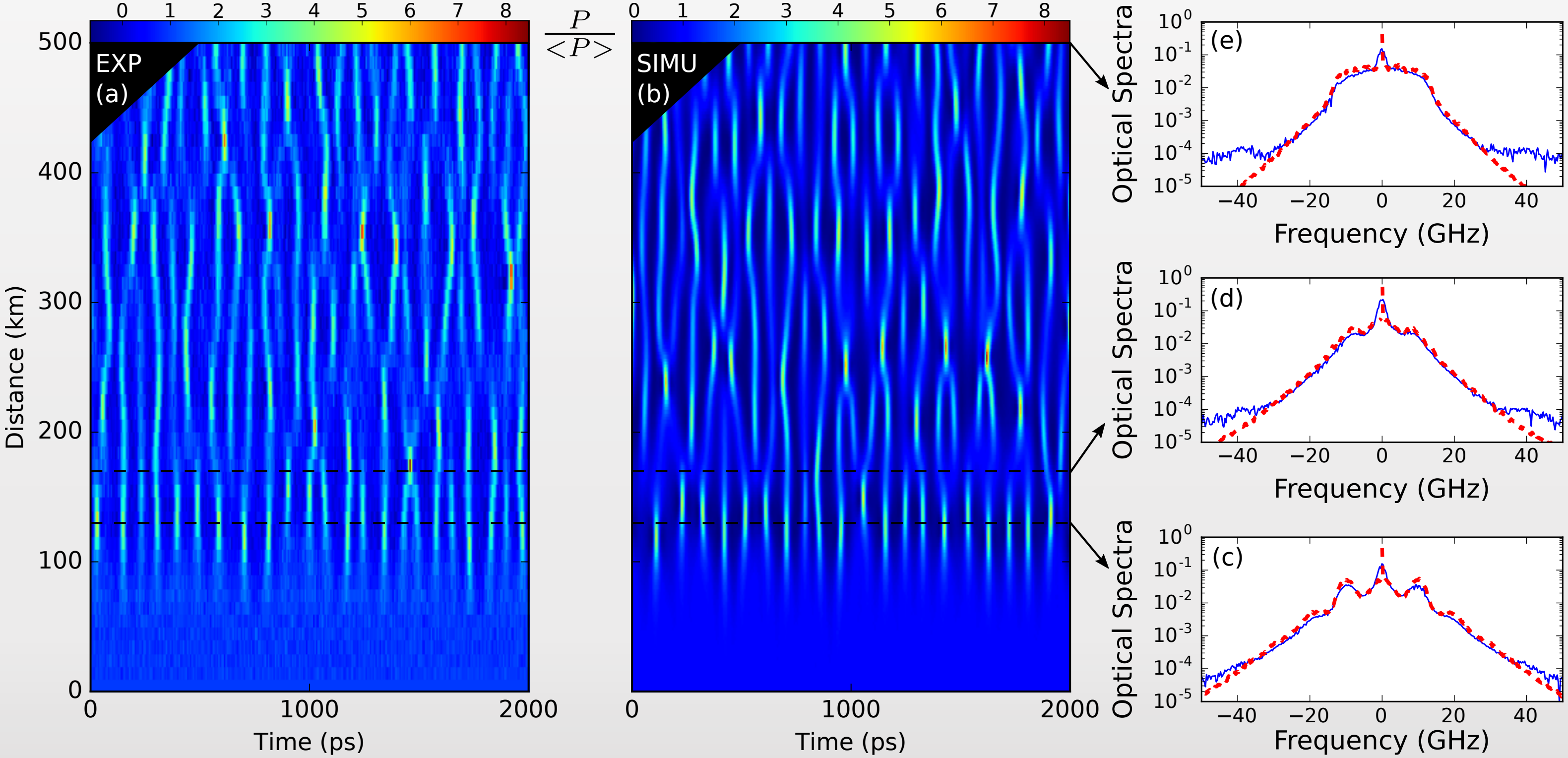


Nonlinear stage of MI : oscillations of $g^{(2)}$ around unity
<-> period of modulation instability
(note : random waves : $g^{(2)} > 1$)

Noise-driven Modulation instability (optical fiber experiments)

A. E. Kraych, D. Agafontsev, S. Randoux, and P. Suret, Phys. Rev. Lett. **123**, 093902 (2019)



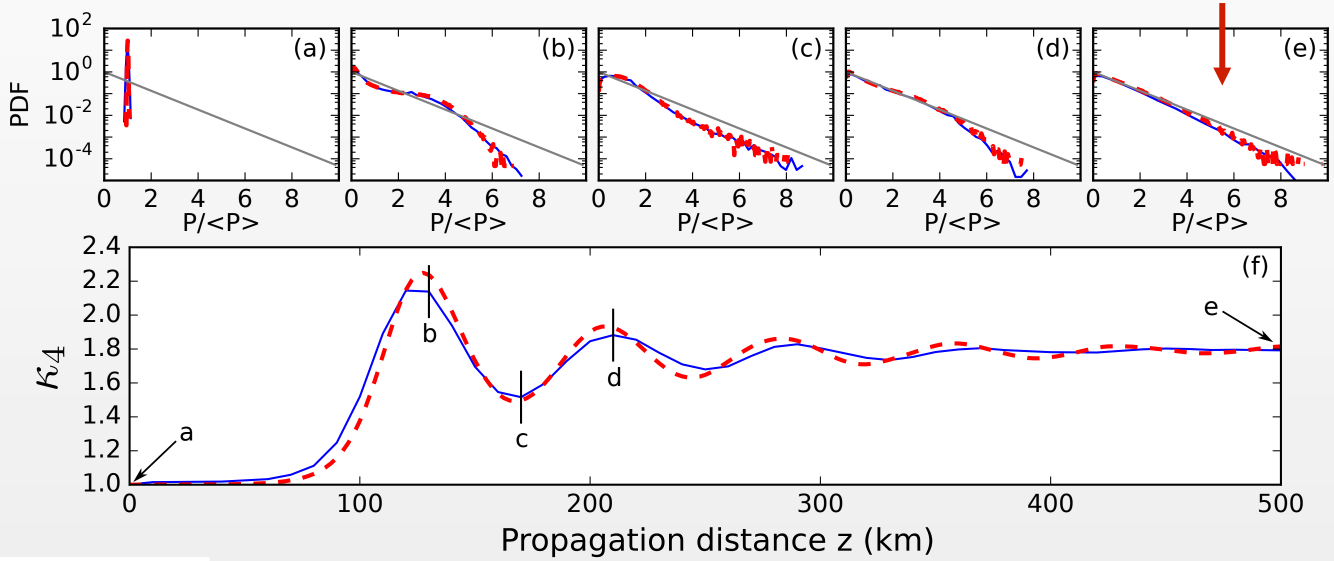


Noise-driven Modulation instability (optical fiber experiments)

A. E. Kraych, D. Agafontsev, S. Randoux, and P. Suret, Phys. Rev. Lett. **123**, 093902 (2019)

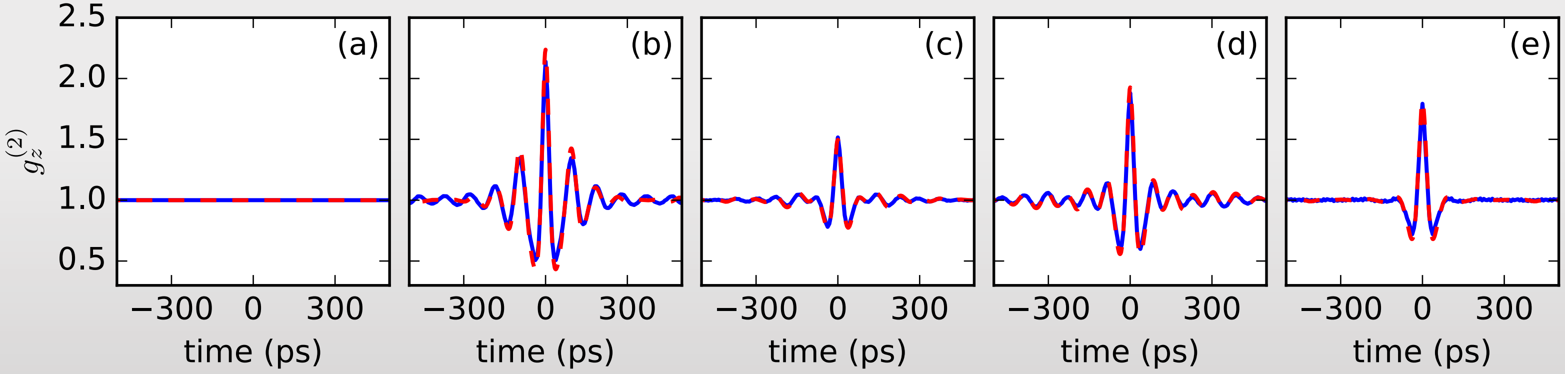
$$P = |\psi|^2$$

Gaussian statistics of ψ



$$\kappa_4(z) = \frac{\langle P(z,t)^2 \rangle}{\langle P(z,t) \rangle^2}$$

$$g_z^{(2)}(\tau) = \frac{\langle P(z,t) P(z,t+\tau) \rangle}{\langle P(z,t) \rangle^2}$$



Noise-driven Modulation instability (water tank experiments)

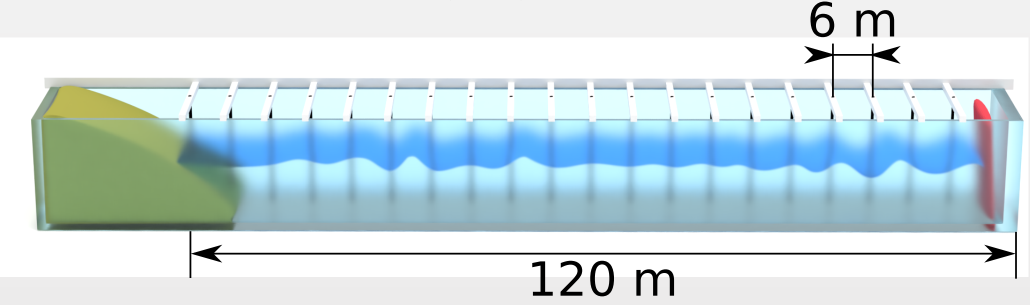
F. Copie, S. Randoux, A. Tikan, P. Suret
Phlam, Univ Lille, France

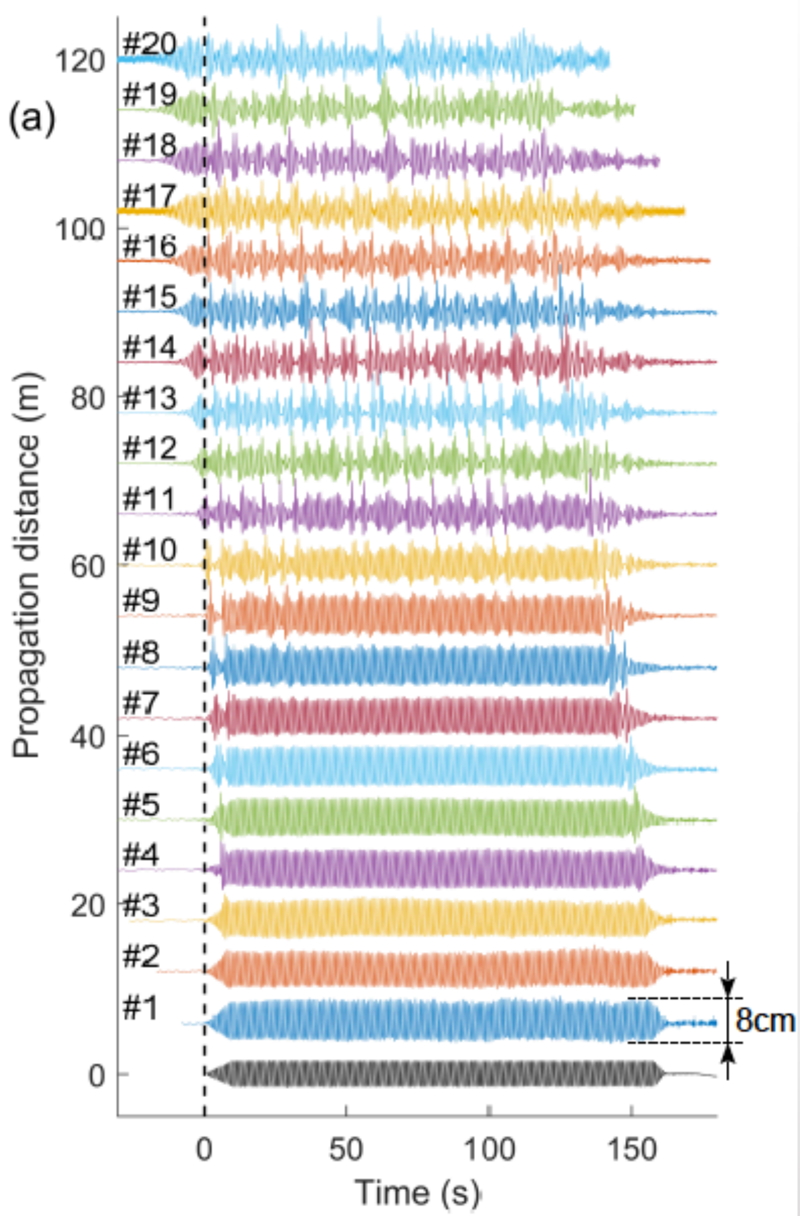
Eric Falcon, Annette Cazaubiel (PhD)
MSC, *Univ. Paris Diderot, France*

Guillaume Michel, Gaurav Prabhudesai (ENS, PhD),
Ecole Norm. Sup., France

Félicien Bonnefoy, Guillaume Ducrozet, *Ecole Centrale de Nantes*

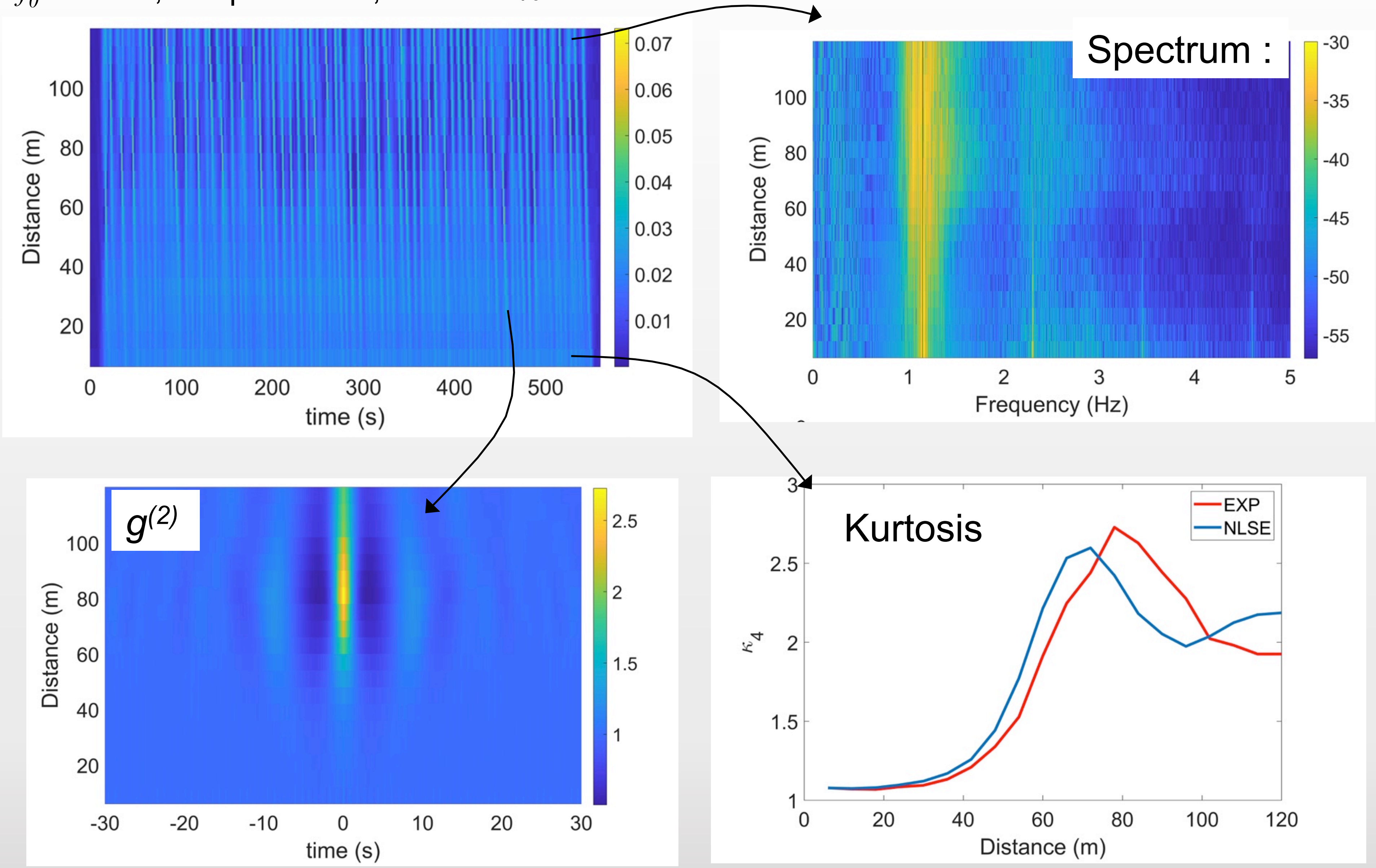
Amin Chabchoub, *Univ. Sydney, Australia*





Noise-driven Modulation instability (water tank experiments)

f_0=1.15Hz, steepness=0.1, noise=10%



Toward an IST theory of integrable turbulence ?

- **Inverse scattering transform (nonlinear Fourier transform)**

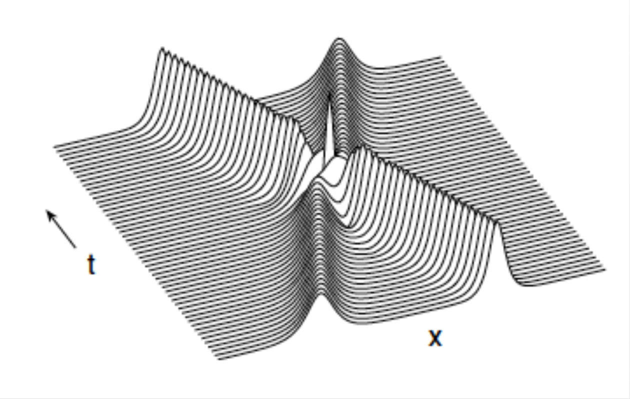
$$i\frac{\partial\psi}{\partial t}+\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}+|\psi|^2\psi=0$$

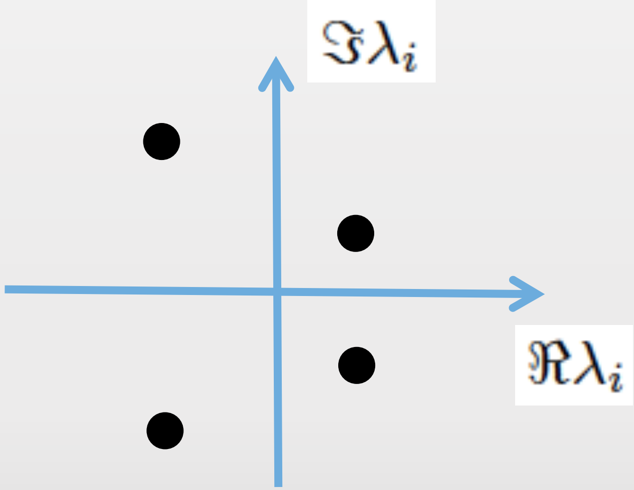
Zero-boundary condition:

continuous and discrete spectrum (solitons) + norming constants

- **N-solitons solutions**

$$\forall n:\lambda_n=\text{const},\quad C_n(t)=C_n(0)e^{-2i\lambda_n^2 t}$$





N ~200

Gelash, A. A., & Agafontsev, D. S., Physical Review E, 98(4), 042210 (2018)

MI modeled by soliton gas ?

Zakharov, V. E., Sov. Phys. JETP, 33(3), 538-540, (1971).
El, G. A., & Kamchatnov, A. M., Phys. Rev. Lett. 95(20), 204101, (2005)

$$i\frac{\partial\psi}{\partial t}+\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}+|\psi|^2\psi=0$$

$$\psi(t=0,x)=1+\eta(x)$$

where η is the noise with $\langle\eta\rangle = 0$ and $\sqrt{\langle|\eta|^2\rangle} \ll 1$

Soliton gas with the Weyl distribution

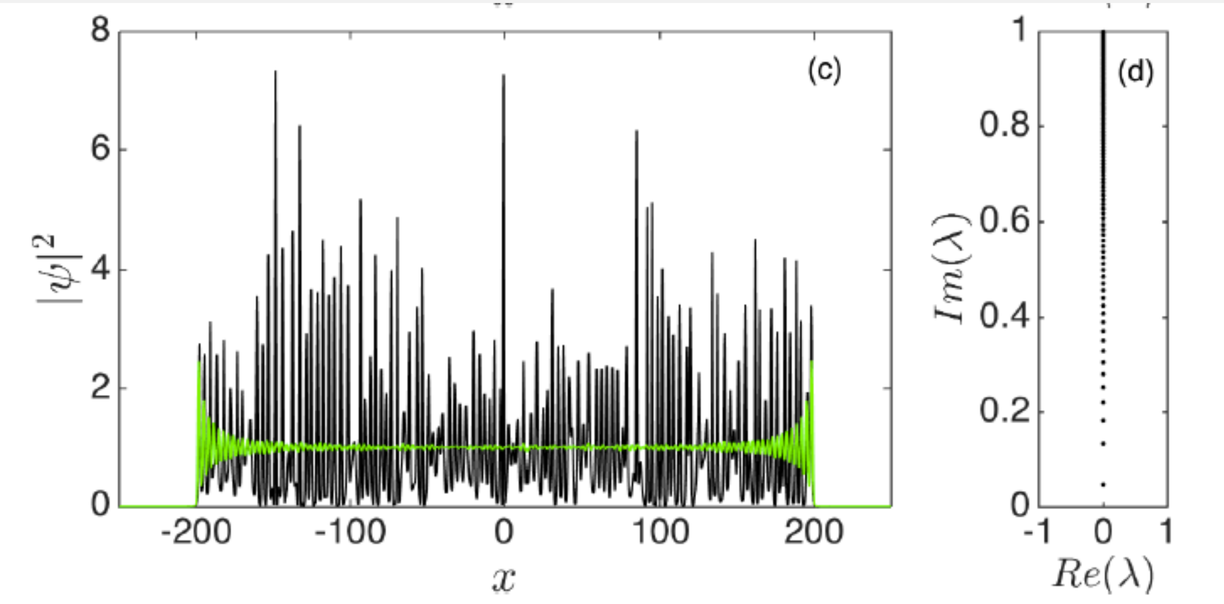
$$f(\lambda)=f(\beta)=\beta/(\sqrt{1-\beta^2}) \text{ where } \lambda = i\,\beta$$

Bound N soliton with :

$$\lambda_n = i\,\beta_n = i\sqrt{1-\left[\frac{\pi(n-\frac{1}{2})}{L_0}\right]^2}, \quad n=1,2,\ldots,N$$

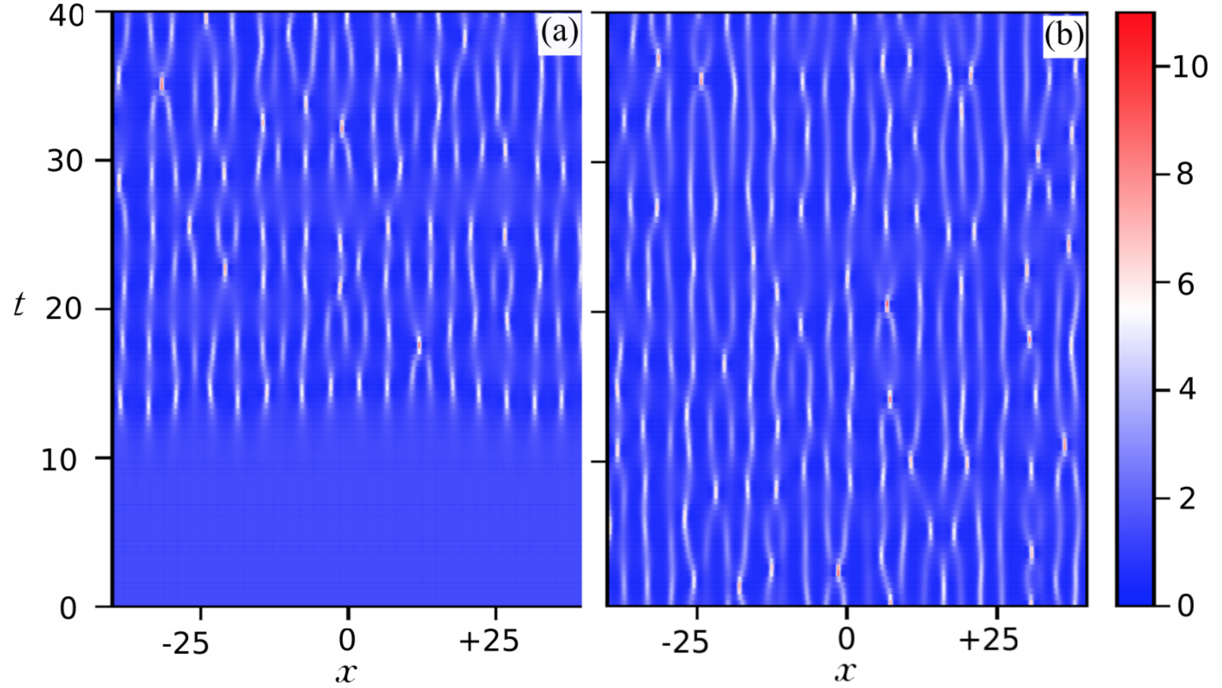
$$N=\text{int}[L/\pi]$$

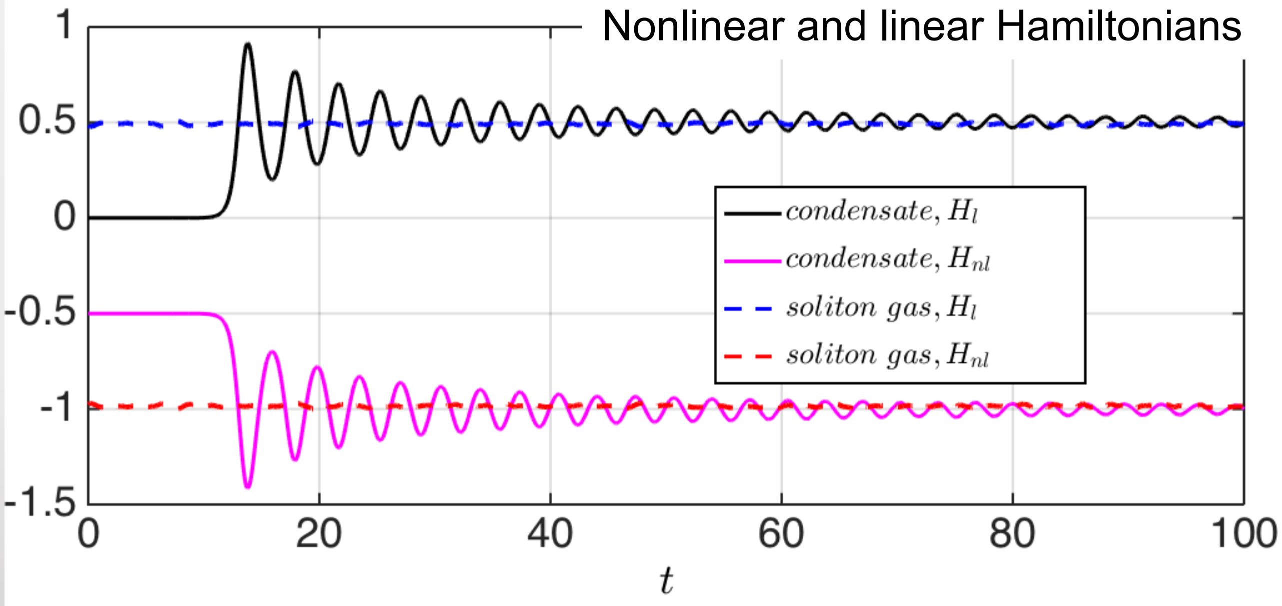
Random phase
of the norming constants



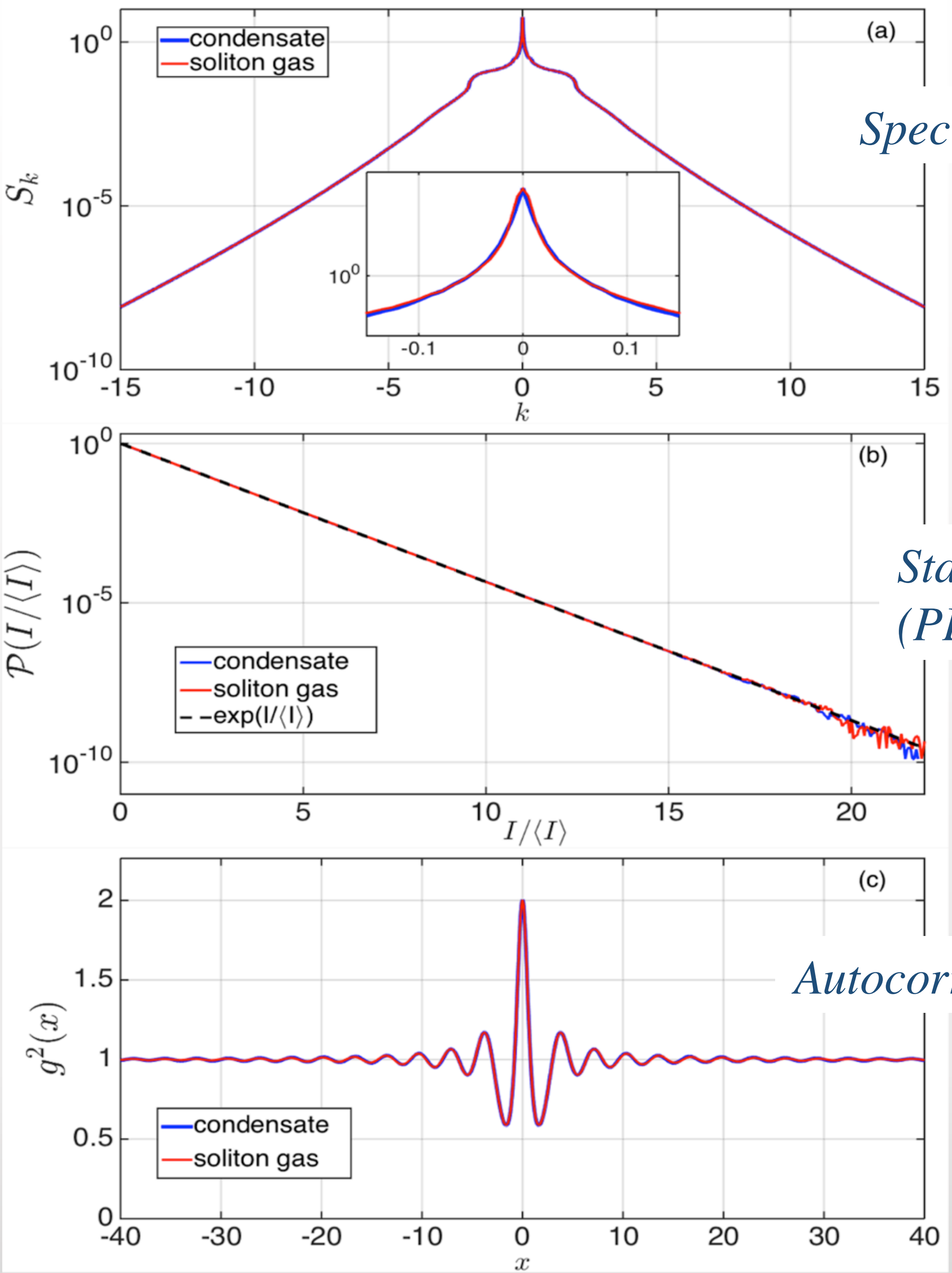
Gelash, A. , Agafontsev, D., Zakharov, V., El, G., Randoux, S., & Suret, P. , accepted for publication in PRL, arXiv, *1907.07914*. (2019)

Nsolitons vs MI




Nonlinear and linear Hamiltonians

Nsolitons vs MI



Spectrum

Statistics (PDF)

Autocorrelation

$$g^{(2)}(x) = \frac{\langle|\psi(t,x')|^2|\psi(t,x'+x)|^2\rangle}{\langle|\psi(t,x')|^2\rangle^2}$$

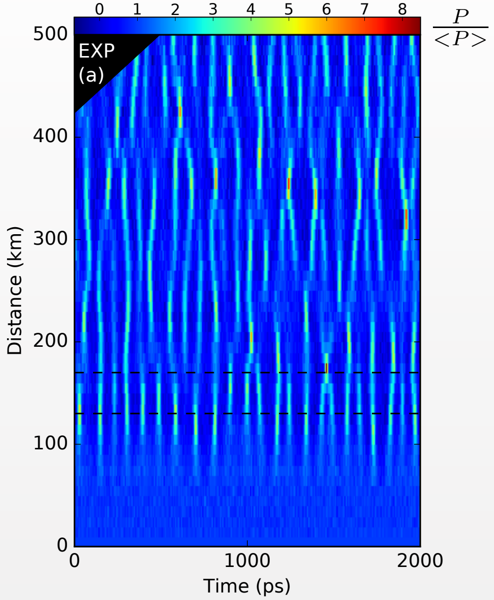
Conclusion and perspectives

Integrable turbulence in optics

✓ Recent experimental breakthrough (ultrafast measurement techniques)

S. Randoux *et al*, Physica D : Nonlinear Phenomena, **333**, (2016)
P. Walczak *et al*, Phys. Rev. Lett., **114**, 143903, (2015)
P. Suret *et al*. Nat. Commun. **7,** 13136 (2016)
A. Tikan, *et al*., Nat. Photon., **12**, 228 (2018)

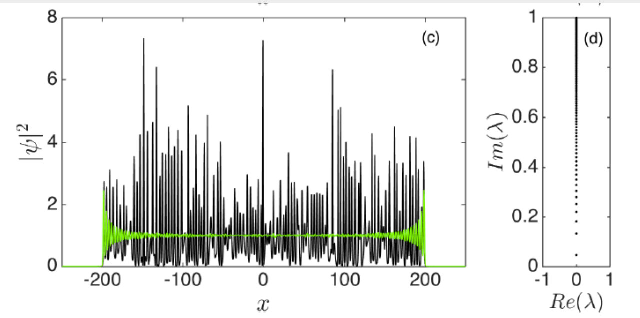
A. E. Kraych, *et al*., Phys. Rev. Lett. **123**, 093902 (2019)



Theory : many open questions !

✓ Wave Turbulence theory A. Picozzi *et al*., Physics Reports, (2014)
✓ Semi-classical approach A. Tikan *et al*., PRL **119**, 033901 (2017)
✓ Soliton Gas: a model of integrable turbulence

Gelash, A. *et al , accepted for publication in PRL, arXiv:1907.07914* (2019)



✓ Discrete spectrum+continuous spectrum ?
✓ Finite gap theory

Pierre Suret, Stéphane Randoux, François Copie
Alexey Tikan (PhD), Rebecca El Koussaifi (PhD), Adrien Kraych (PhD), Alexandre Lebel (PhD)
Christophe Szwaj, Clément Evain, Serge Bielawski
Laboratoire de Physique des Lasers, Atomes et Molécules (Phlam), Univ. de Lille, France

Gennady El, Thibault Congy (Postdoc), Giacomo Roberti (PhD), *Northumbria univ., UK*
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Vladimir Zakharov, *Landau Institute for Theoretical Physics, Chernogolovka, Russia*
Antonio Picozzi, *Dijon, France*

Miguel Onorato, *Torino, Italy*
Eric Falcon, Annette Cazaubiel (PhD) MSC, *Univ. Paris Diderot, France*
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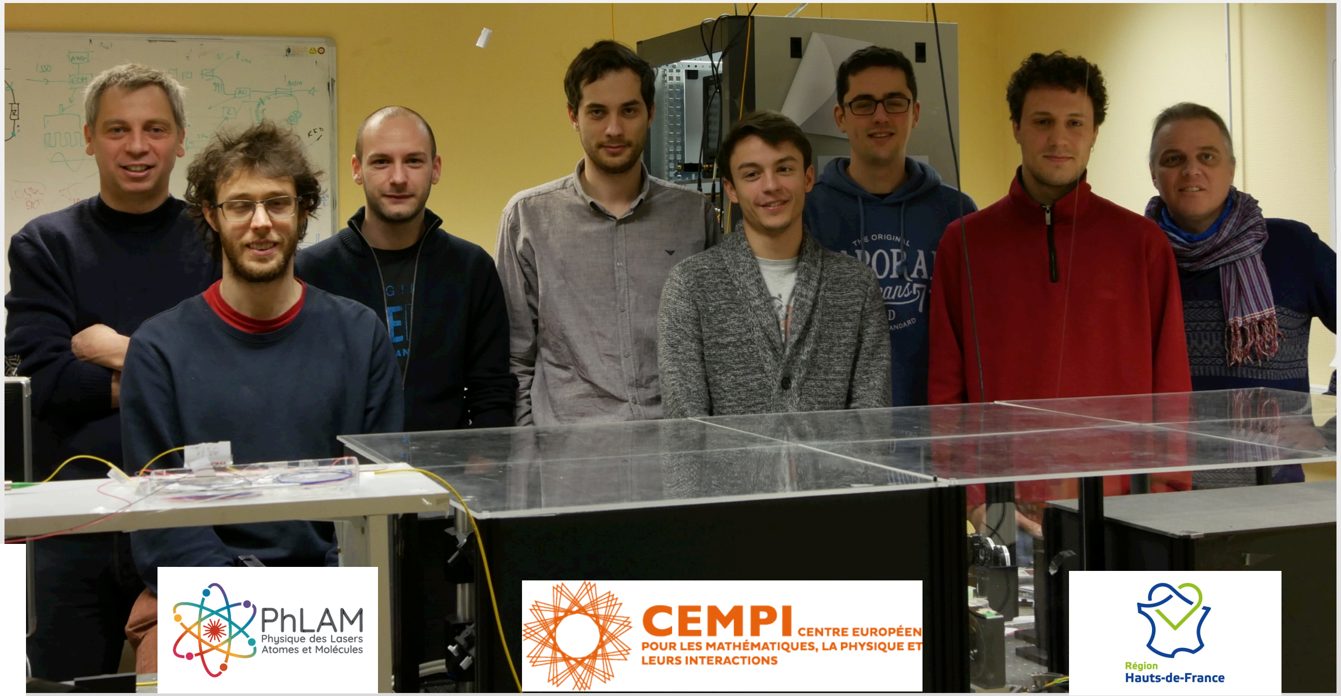

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