Wave amplification phenomena and a novel wave-energy converter

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Outline

- Introduction
- Bore-soliton splash

  Experimental setup and results

  Mathematical model and simulations
- Novel wave-energy device

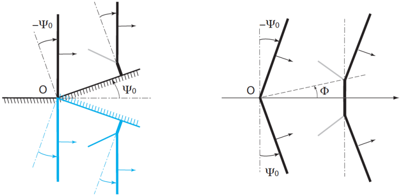
  Proof-of-concept

  Modelling

  Simulations
- Discussion

Motivation

- John Scott Russell (1834): discovered "wave of elevation" Korteweg & De Vries (1895): sech–wave soliton solution
- Public outreach event that creates simple solitons but also leads to an extreme wave
- Idea of creating a rectangular channel with a contraction, to generate rogue-wave effects



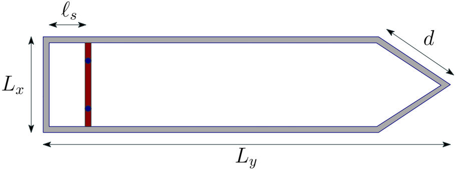
Rogue waves: introduction

- Rogue waves: anomalously high-amplitude waves

$$\mathrm{AI} = \frac{H_{rw}}{H_s} > 2$$

- They occur rarely and are difficult to predict
- Different types: spatial wave focussing, episodic waves (e.g. tsunamis), crossing seas with pyramidal waves
- Understanding their occurrence is relevant to maritime and coastal engineering

Experimental setup



Wavetank length	$L_y = 43.63 \pm 0.1$ m
Wavetank width	$L_x = 2$ m
Wavetank height	$L_z = 1.2$ m
Contraction length	$d = 2.7$ m
Location of sluice gate	$\ell_s = 2.63$ m
Rest-water level (high)	$h_1 = 0.9$ m
Rest-water level (low)	$h_0 = 0.43$ m
Sluice-gate thickness	$y_2 - y_1 = 10$ cm
Sluice-gate release speed	$V_g \approx 2.5$ m/s
Sluice-gate removal time	$T_s = h_1/V_g \approx 0.36$ s

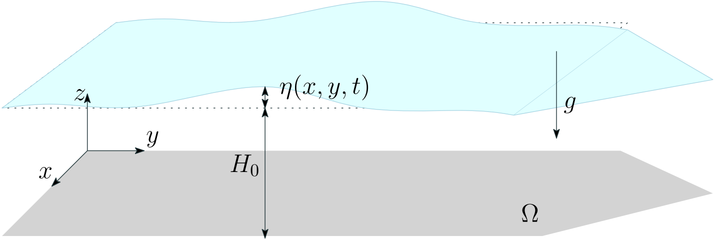
Experiments

Trials to establish the highest bore-soliton-splash (BSS): $\mathrm{AI} \approx 10$

Case	h_0 (m)	h_1 (m)	H_s (m)	H_{rw} (m)	Comments
1	0.32	0.67	-	0.6	bore
2	0.38	0.74	-	2.5	good splash
3	0.41	0.9	0.35	3.25	thin jet (cf. 6 & 8)
4	0.47	1.0	0.35	1	bore & low
5	0.41	1.02	0.40	1.5	bore & low
6	0.41	0.9	0.35	3.5	BSS (cf. 3 & 8)
7	0.45	0.8	0.35	2.5	good splash
8	0.41	0.9	0.35	3.5	BSS & ++
9	0.43	0.9	0.45	1.8	collapsing into sheets

Mathematical modelling

- Potential flow: three-dimensional velocity $\boldsymbol{u}$ approximated using a velocity potential $\phi = \phi(x, y, z, t)$ as $\boldsymbol{u} = \boldsymbol{\nabla}\phi$
- Free surface at $z = h(x, y, t) = H_0 + \eta(x, y, t)$ over a flat bottom at $z = 0$



Variational principle[1] (VP)

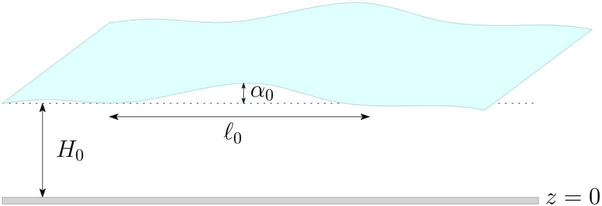
$$
\begin{aligned}
0 &= \delta \int_0^T \mathcal{L}[\phi, \eta]\,\mathrm{d}t \\
&= \delta \int_0^T \iint_\Omega \int_0^{H_0+\eta} \Big(\partial_t \phi + \frac{1}{2} |\boldsymbol{\nabla} \phi|^2 + g(z - H_0 - \eta_R) \Big)\,\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}t
\end{aligned}
$$

Potential flow water-wave equations

$$
\begin{aligned}
\boldsymbol{\nabla}^2 \phi = 0 &\quad \text{in} \quad \Omega \\
\partial_t \eta + \nabla\phi \cdot \nabla\eta - \partial_z \phi = 0 &\quad \text{at} \quad z = H_0 + \eta \\
\partial_t \phi + \frac{1}{2} |\boldsymbol{\nabla}\phi|^2 + g(\eta - \eta_R) = 0 &\quad \text{at} \quad z = H_0 + \eta \\
\mathbf{n} \cdot \nabla\phi = 0 &\quad \text{on} \quad z = 0 \ \text{ and } \ \partial\Omega
\end{aligned}
$$

[1] Luke (1967) *J. Fluid Mech.*

Benney-Luke approximation



$$\epsilon = \alpha_0 / H_0 \ll 1$$
$$\mu = (H_0/\ell_0)^2 \ll 1$$

$$(x, y) = \frac{H_0}{\sqrt{\mu}} (\hat{x}, \hat{y}), \quad z = H_0 \, \hat{z}, \quad t = \frac{H_0}{\sqrt{g H_0 \mu}} \, \hat{t}, \quad \phi = \epsilon H_0 \sqrt{\frac{g H_0}{\mu}} \, \hat{\phi}, \quad \eta = \epsilon H_0 \, \hat{\eta}$$

Expansion about $\Phi(x, y, t) = \phi(x, y, z = 0, t)$ in powers of μ[1]

$$\phi = \Phi - \frac{\mu}{2} z^2 \nabla^2 \Phi + \frac{\mu^2}{24} z^4 \nabla^4 \Phi + \cdots$$

[1] Pego & Quintero (1999) *Physica D*

Benney-Luke approximation

$$0 = \delta \int_0^T \iint_\Omega \eta \partial_t \Phi + \frac{\mu}{2} \nabla \eta \cdot \partial_t \nabla \Phi + \frac{1}{2}(1+\epsilon\eta)|\nabla \Phi|^2 + \frac{1}{2}\eta^2 - \eta_R \eta$$
$$+ \mu \left(\nabla q \cdot \nabla \Phi - \frac{3}{4} q^2 \right) \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}t$$

Equations of motion

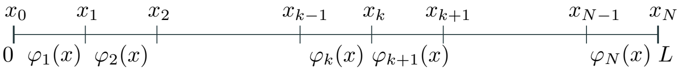
$$\delta\eta: \quad \partial_t \Phi - \frac{\mu}{2} \partial_t \Delta \Phi + \frac{\epsilon}{2} |\nabla \Phi|^2 + \eta - \eta_R = 0$$
$$\delta\Phi: \quad \partial_t \eta - \frac{\mu}{2} \partial_t \Delta \eta + \nabla \cdot \big((1+\epsilon\eta) \nabla \Phi \big) + \mu \Delta q = 0$$
$$\delta q: \quad q = -\frac{2}{3} \Delta \Phi$$

Note: Auxiliary variable q used to lower spatial derivatives order[1]

[1] Bokhove & Kalogirou (2016) *LMS Lecture notes, CUP*

Finite Element implementation

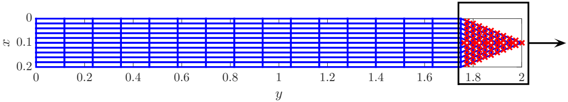
- Continuous Galerkin finite element method (FEM)

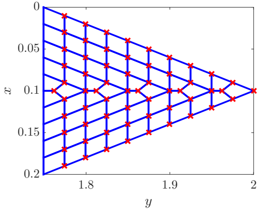


$$\eta(x,y,t) \approx \eta_h(x,y,t) = \sum_k \eta_k(t)\varphi_k(x,y) \quad \text{etc.}$$

with $k = 1, \ldots, N$ over all nodes on a quadrilateral mesh[1]

- Symplectic 2nd-order Störmer-Verlet scheme for time discretisation
- Algebraic VP conserves discrete energy $\Rightarrow$ accurate and robust

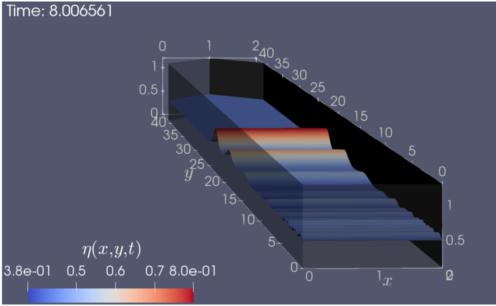




[1] Kalogirou & Bokhove (2016) *Proceedings of OMAE2016*

Simulation

Experiment vs. Simulation



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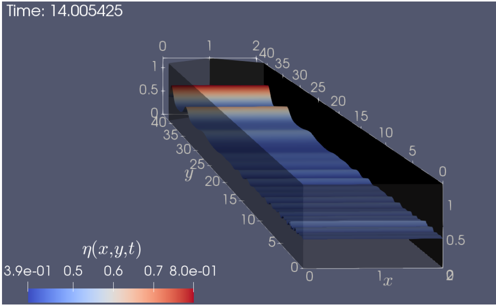

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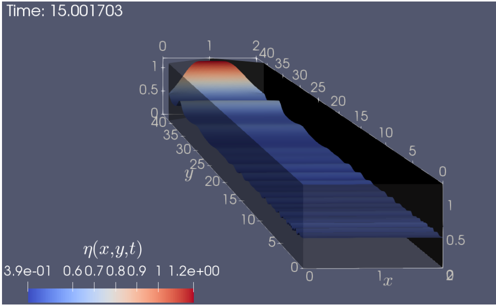
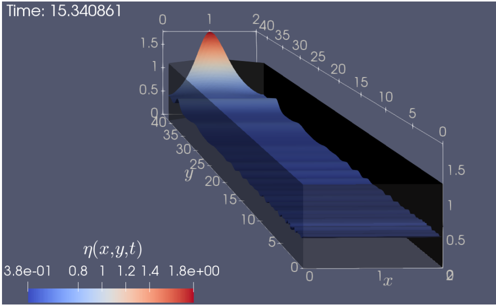
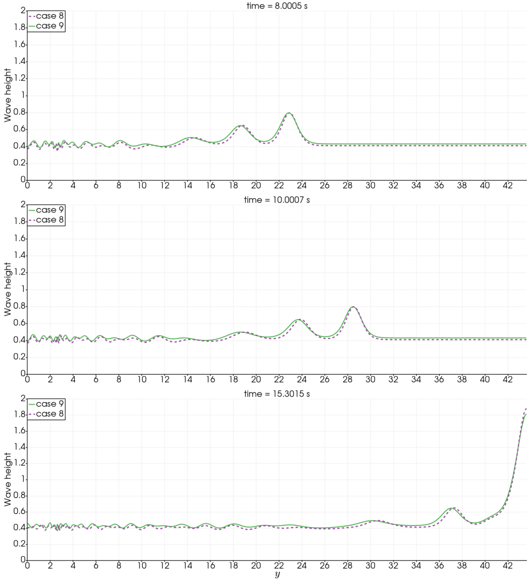


Experiment vs. Simulation

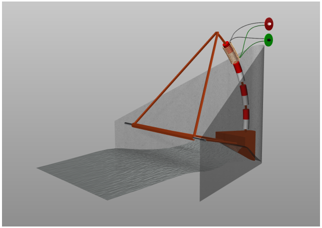


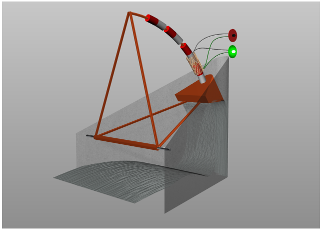
Comparison between the two cases

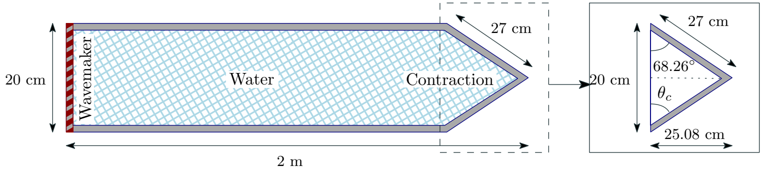


- Case 8: reduction of wave amplitude and speed due to wave breaking
- The wave dispersion in Benney-Luke equations is too strong
- A different model is needed

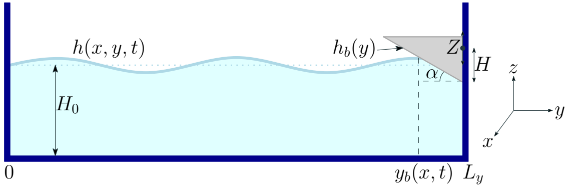
Wave-energy device: proof-of-concept







The buoy motion can be described by the position of its centre of mass $Z = Z(t)$ and corresponding velocity $W = W(t) = \mathrm{d}Z/\mathrm{d}t$

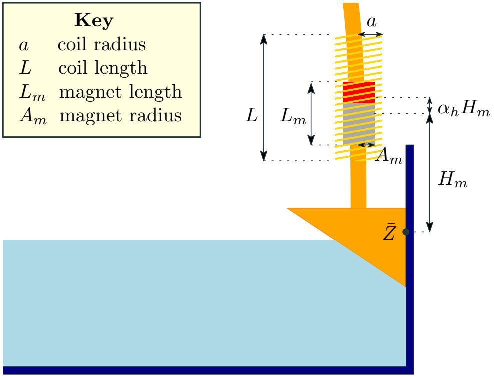


$$\text{Buoy hull:} \qquad z = h_b(y;t) = Z(t) - H - \tan\alpha\,(y - L_y)$$

$$\text{Constraint:} \qquad h(x, y, t) = h_b(y;t) \quad \text{for} \quad y \geq y_b(x, t)$$

Magnetic-induction actuator

A novel model of the magnetic-induction actuator is developed by using the Maxwell's equations in a thin-wire approximation.



- Coil inductance L_i
- Current $I = I(t)$ and electrical charge $Q = Q(t)$, with $I = \dot{Q}$
- Conjugate momentum of current $P_Q = L_i\dot{Q} - K(Z)$, with $K'(Z) = \gamma G(Z)$ depending on properties of mast (length), magnet (magnetic dipole, length, radius) and coils (placement, length, radius)

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Monolithic variational principle of the entire, coupled water-wave problem (shallow), buoy motion and magnetic-induction actuator

Equations of motion

$$\partial_t h + \boldsymbol{\nabla} \cdot (h \boldsymbol{\nabla} \phi) = 0$$

$$\partial_t \phi + \frac{1}{2} |\boldsymbol{\nabla} \phi|^2 + g(h - H_0) + \lambda \Theta(y) = 0$$

$$h - h_b = 0 \quad \text{for} \quad y \geq y_b(x,t)$$

$$\dot{Z} = W$$

$$M\dot{W} + Mg + \frac{K'(Z)}{L_i}\big(P_Q + K(Z)\big) - \rho \iint_\Omega \lambda \Theta(y) \, \mathrm{d}x \, \mathrm{d}y = 0$$

$$\dot{Q} = \frac{\big(P_Q + K(Z)\big)}{L_i} \equiv I$$

$$\dot{P}_Q = -(R_c + R_i)I - V_s$$

Power output & linearisation

- Voltage across the LEDs is given by Shockley equation

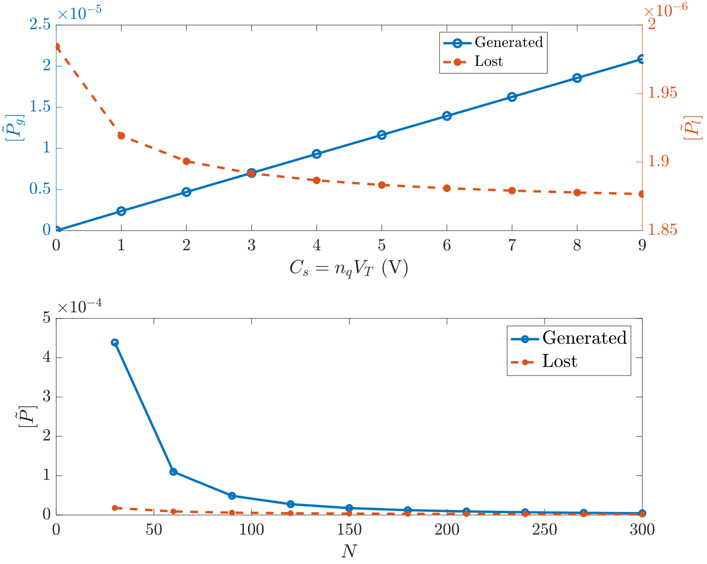
$$V_s = n_q V_T \,\mathrm{sign}(I) \ln\left(\frac{|I|}{I_{sat}} + 1\right)$$

- To model losses in the circuit and LED diodes, add wiring resistance R_i and coil resistance R_c
  $\rightsquigarrow$ damping is added a posteriori to the model
- Electrical power output: $P(t) = I(t)V_s(t)$
- The model is simplified by linearising around the rest state:

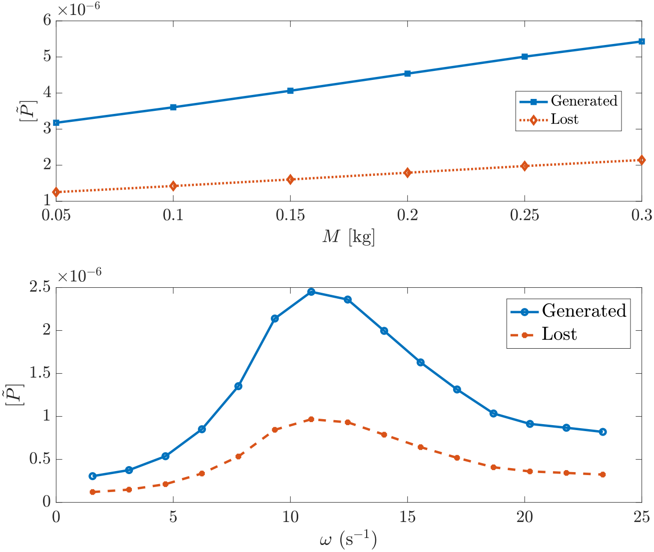
$$\begin{gathered}\phi(x,y,z,t)=0, \quad h(x,y,t)=H(x,y), \quad \lambda(x,y,t)=\Lambda(x,y),\\ y_b(x,t)=L_b, \quad Z(t)=\bar{Z}, \quad W(t)=0,\\ Q(t)=0, \quad P_Q(t)=-K(\bar{Z}), \quad I(t)=0\end{gathered}$$

Preliminary simulations

Power generation



Power generation



Summary

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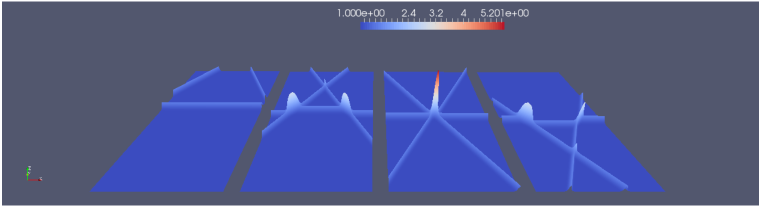
- Bore-soliton splash
- Wave-energy converter
- Variational math. model
- Bespoke FEM simulations
- Optimisation of WEC?

$\rightsquigarrow$ O. Bokhove, A. Kalogirou and W. Zweers (2019) From bore-soliton-splash to a new wave-to-wire wave-energy model. *Water Waves*, to appear.

Applications

- Relation to rogue waves in crossing seas
- Oblique interaction of two solitary waves[1]
- Exact web-soliton solutions of the KP equation[2]

$$\left(2\eta_\tau + 3\eta\eta_X + \frac{1}{3}\eta_{XXX}\right)_X + \eta_{YY} = 0$$



[1] Gidel, Bokhove & Kalogirou (2017), *Nonlin. Proc. Geoph.*

[2] Baker (2017), *MSc thesis, Leeds Uni.*; Gidel (2018), *PhD thesis, Leeds Uni.*

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