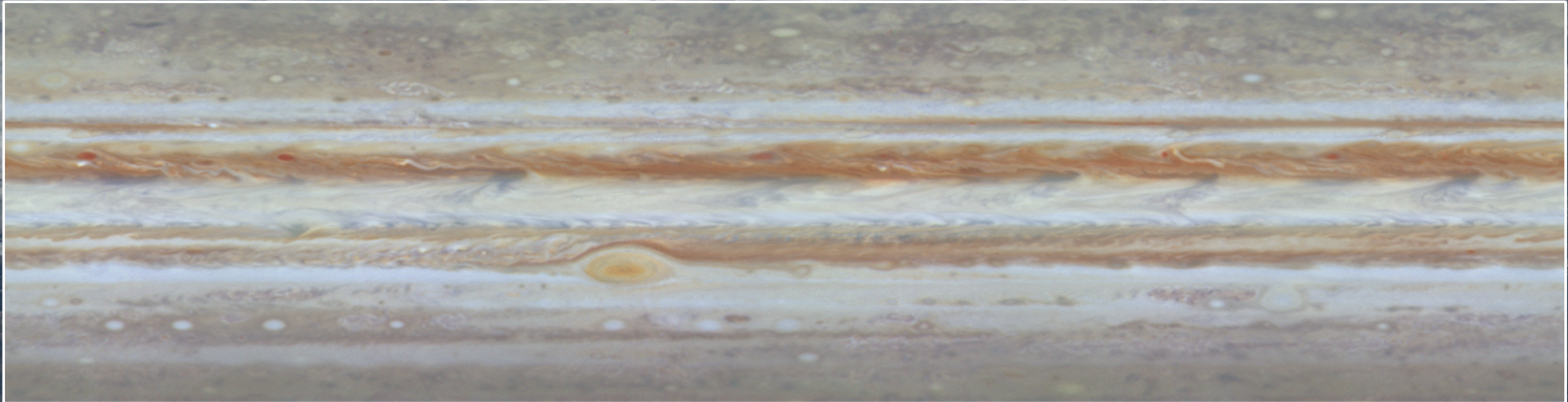


Variability of Stochastically Forced Zonal Jets

Laura Cope, Peter Haynes

Department of Applied Mathematics and Theoretical Physics, University of Cambridge



MOTIVATION

Gaseous Giant Planets



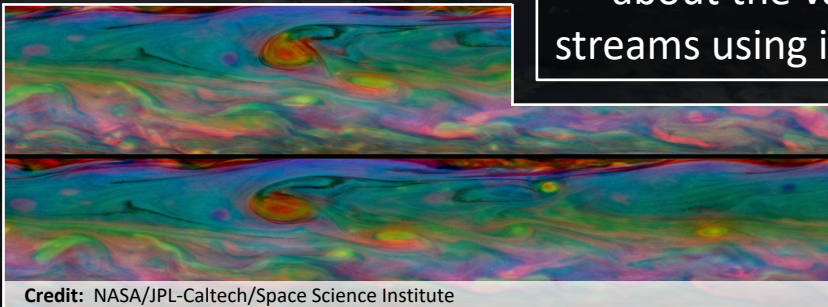
Credit: NASA, ESA

Earth's Atmosphere & Oceans

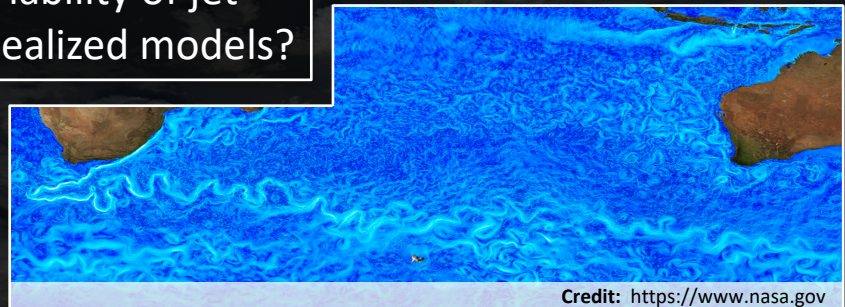


Credit: NASA / Science Photo Library

What insights can we learn
about the variability of jet
streams using idealized models?



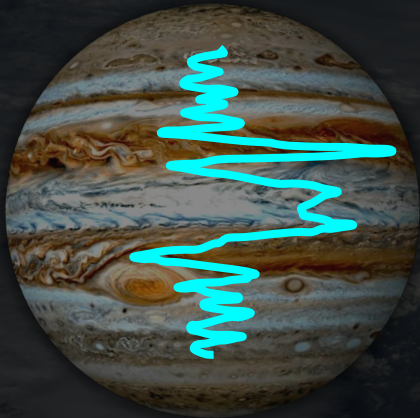
Credit: NASA/JPL-Caltech/Space Science Institute



Credit: <https://www.nasa.gov>

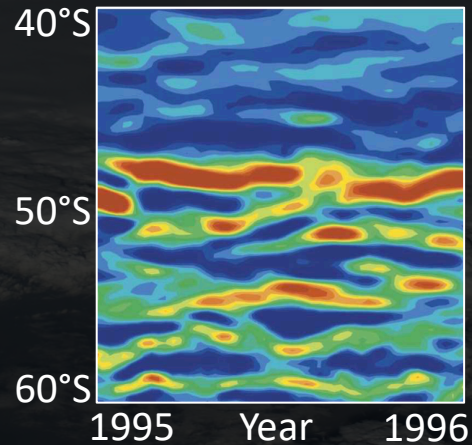
OBSERVATIONS

Jupiter



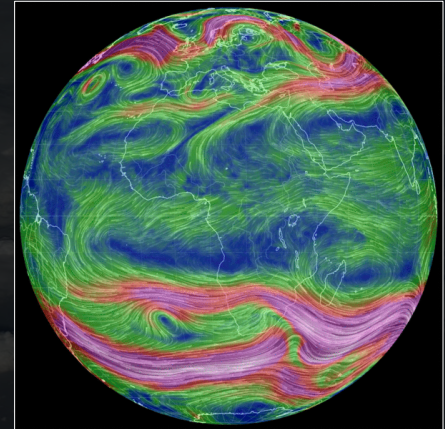
Voyager 1979-1980

Earth's Oceans



(Sokolov, Rintoul 1996)

Earth's Atmosphere



(earth.nullschool.net)

Increasing time variability →

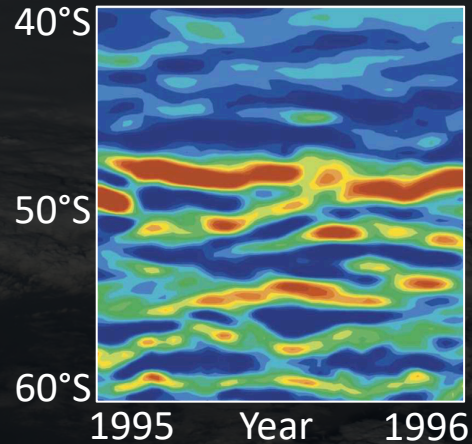
OBSERVATIONS

Jupiter



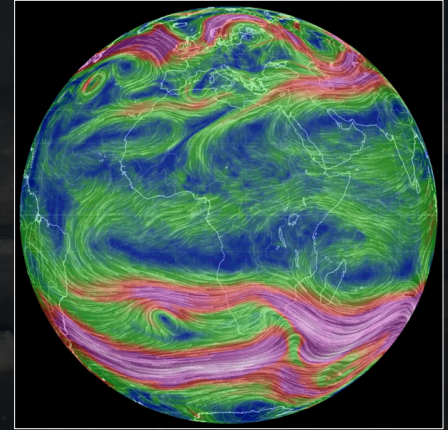
Voyager 1979-1980
Cassini 2000

Earth's Oceans



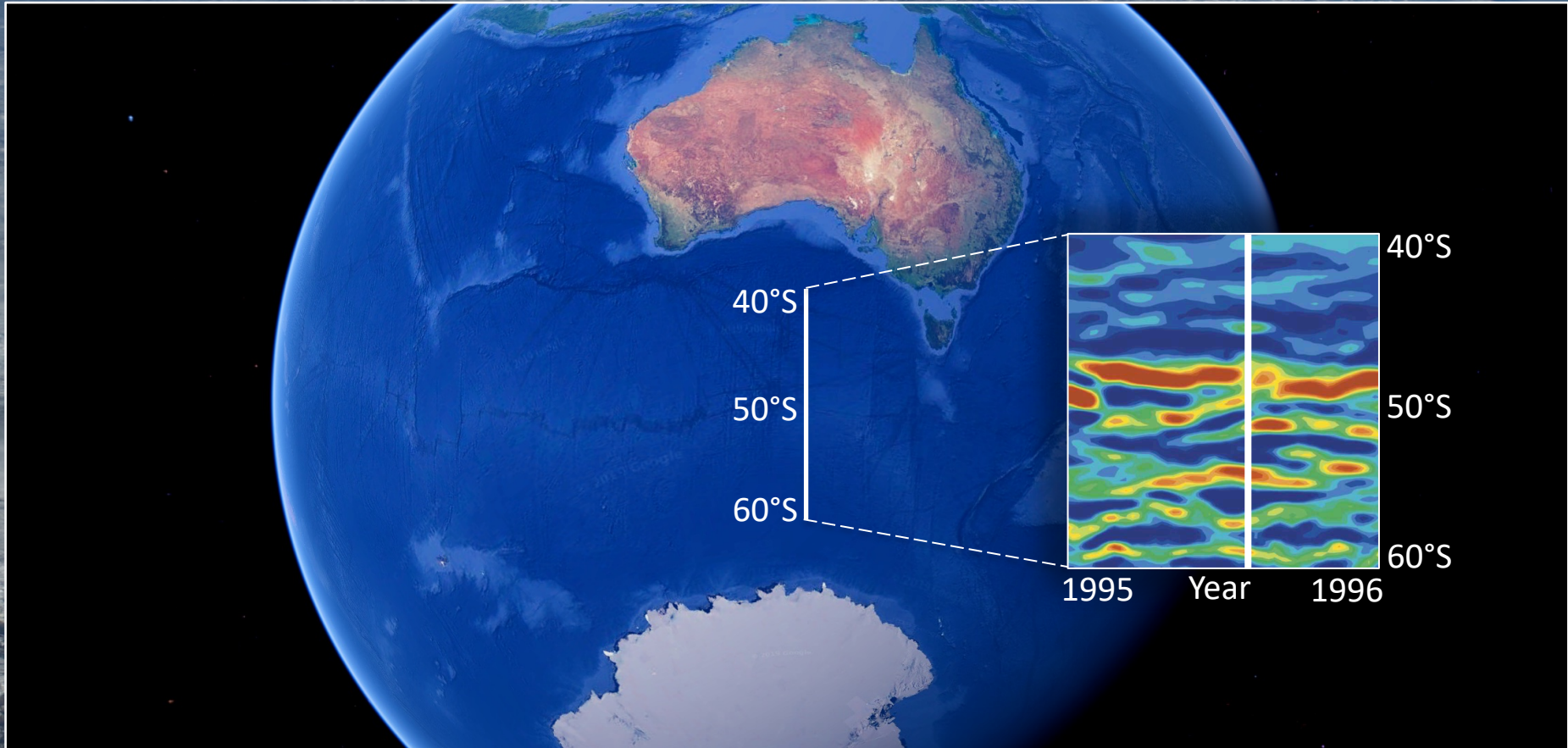
(Sokolov, Rintoul 1996)

Earth's Atmosphere

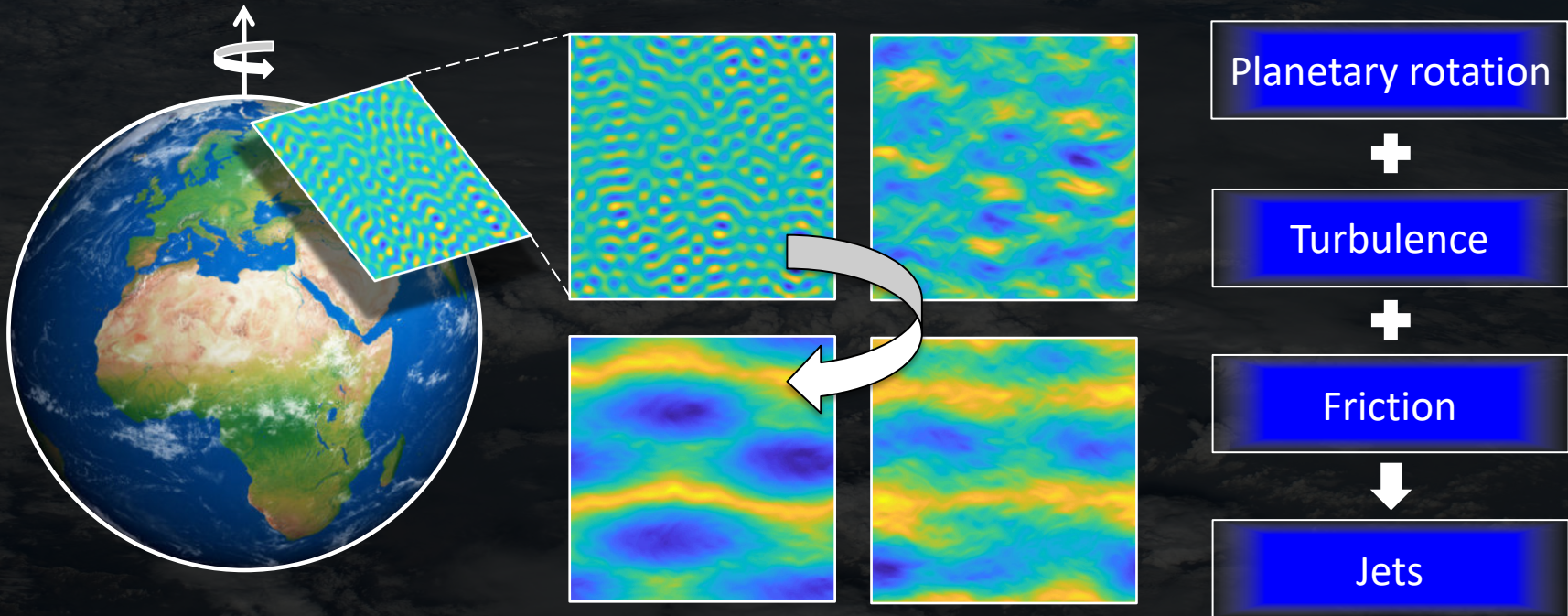


(earth.nullschool.net)

Increasing time variability →



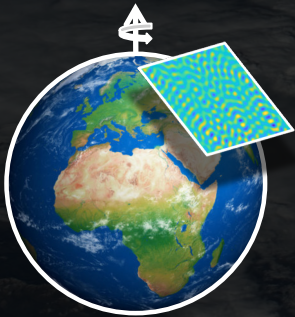
OVERVIEW OF IDEALIZED MODELS



IDEALIZED MODELS: MATHEMATICAL FORMULATION

Features

2D (barotropic)
 Doubly periodic
 Beta plane
 Stochastic force



Equation of Motion: Vorticity Equation

Planetary Rotation

Stochastic forcing

Linear friction

Hyper-viscosity

$$\frac{\partial \zeta}{\partial t} + \mathbf{u} \cdot \nabla \zeta + \beta v = \xi - \mu \zeta + \nu_n \nabla^{2n} \zeta$$

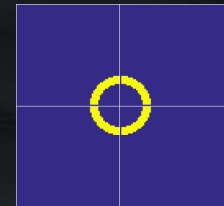
Beta, β

Energy input rate, ϵ

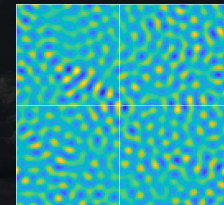
Damping rate, μ

Stochastic Force

Fourier space:



Physical space:



IDEALIZED MODELS: MATHEMATICAL FORMULATION

Generalized
Quasilinear
Approximation

Reference:
Marston, Chini,
Tobias (2016)



Increasing zonal
wavenumber

IDEALIZED MODELS: MATHEMATICAL FORMULATION

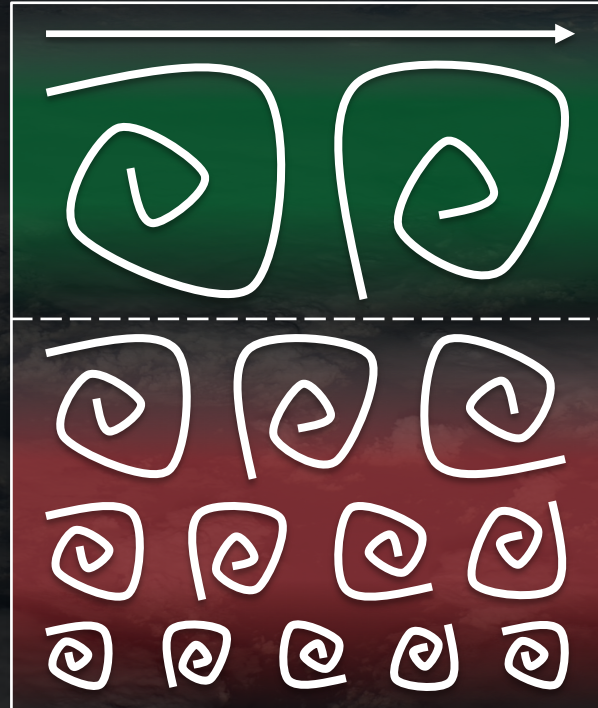
Generalized Quasilinear Approximation

Reference:
Marston, Chini,
Tobias (2016)

Low modes $\leq \Lambda$

Separation = Λ

High modes $> \Lambda$



Increasing zonal wavenumber

IDEALIZED MODELS: MATHEMATICAL FORMULATION

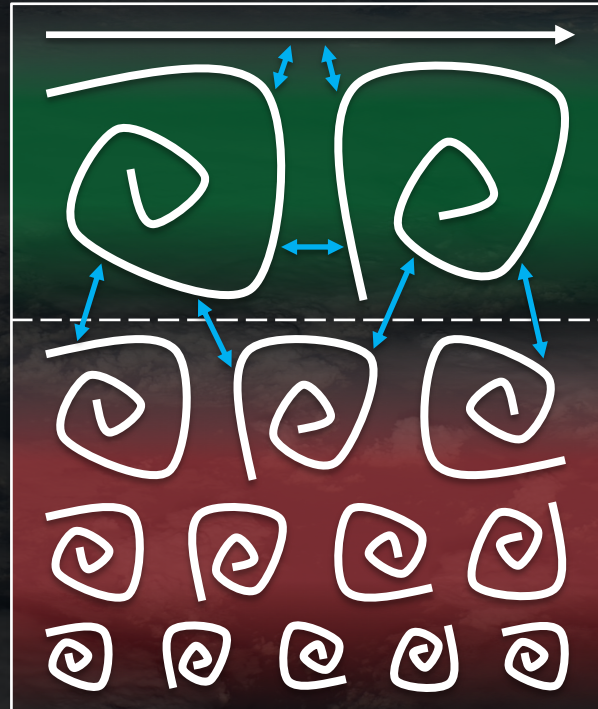
Generalized Quasilinear Approximation

Reference:
Marston, Chini,
Tobias (2016)

Low modes $\leq \Lambda$

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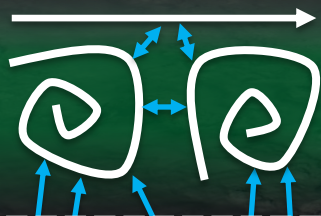


Increasing zonal wavenumber

IDEALIZED MODELS: MATHEMATICAL FORMULATION

Overview

Low modes $\leq \Lambda$



High modes $> \Lambda$



Generalized Quasilinear Approximation

Low-high mode
decomposition:

$$\psi = \sum_{|m| \leq \Lambda} e^{imx} \widehat{\psi}_m + \sum_{|m| > \Lambda} e^{imx} \widehat{\psi}_m = \bar{\psi} + \psi'$$

Basic vorticity
equation:

$$\frac{\partial \zeta}{\partial t} = \mathcal{L}[\zeta] + \mathcal{N}[\zeta, \zeta]$$

Low modes:

$$\frac{\partial \bar{\zeta}}{\partial t} = \mathcal{L}[\bar{\zeta}] + \bar{\mathcal{N}}[\bar{\zeta}, \bar{\zeta}] + \bar{\mathcal{N}}[\zeta', \zeta']$$

High modes:

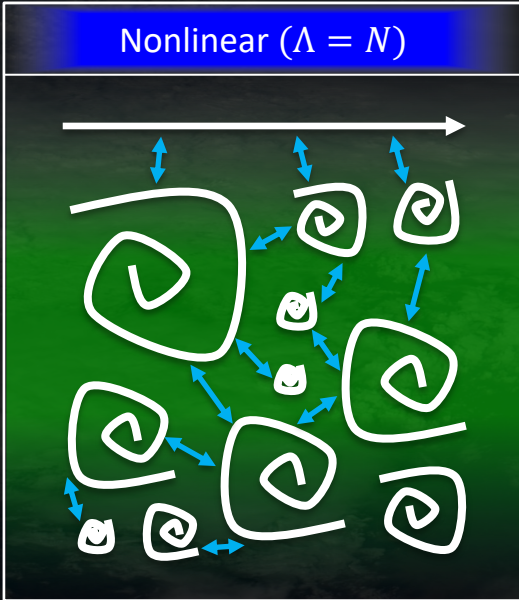
$$\frac{\partial \zeta'}{\partial t} = \mathcal{L}[\zeta'] + \mathcal{N}'[\bar{\zeta}, \zeta'] + \text{[HHNL]}$$

Vorticity
equation:

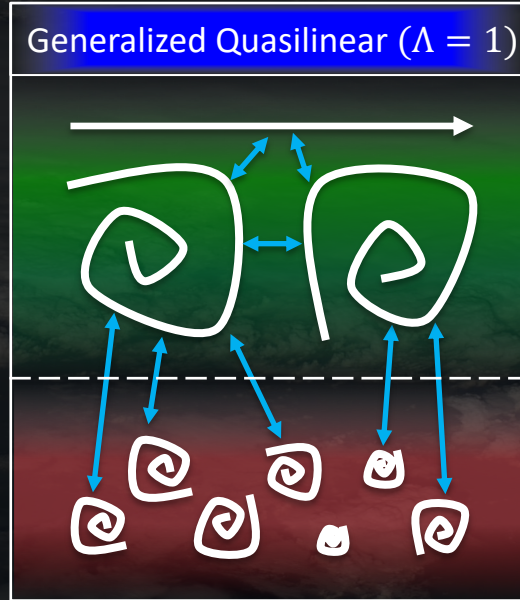
$$\frac{\partial \zeta}{\partial t} + \mathbf{u} \cdot \nabla \zeta + \beta v - \text{[HHNL]} = \xi - \mu \zeta + \nu_n \nabla^{2n} \zeta$$

SUMMARY OF IDEALIZED MODELS

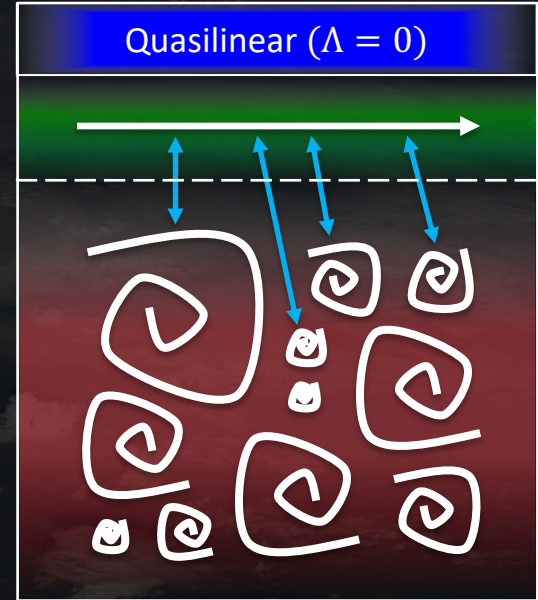
Nonlinear ($\Lambda = N$)



Generalized Quasilinear ($\Lambda = 1$)



Quasilinear ($\Lambda = 0$)

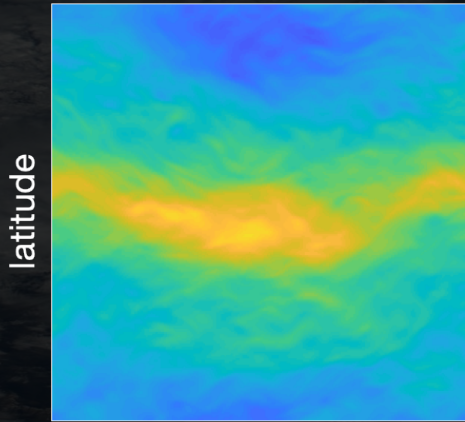


Reduction in nonlinearity



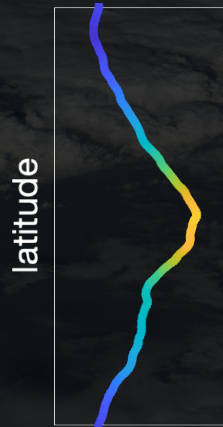
NUMERICAL SIMULATIONS – NONLINEAR (NL) MODEL

Zonal velocity
field



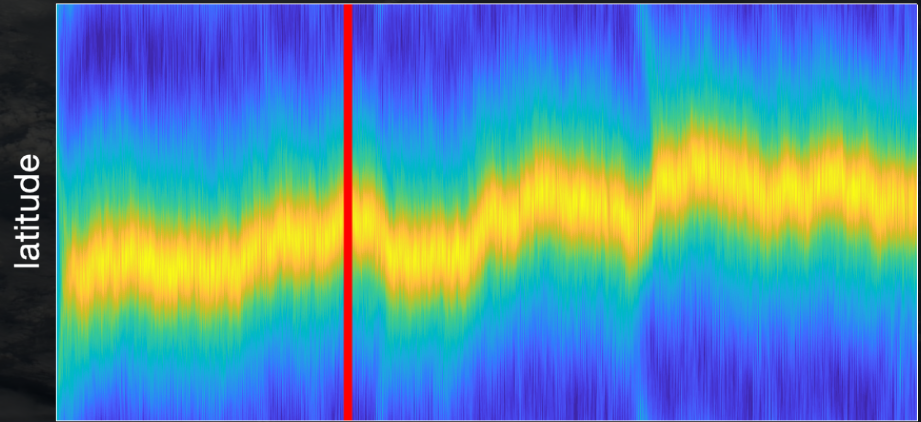
longitude

Zonal mean
zonal velocity



jet profile

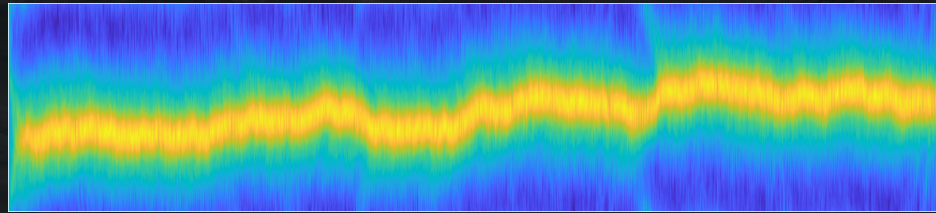
Zonal mean zonal velocity
evolution in time



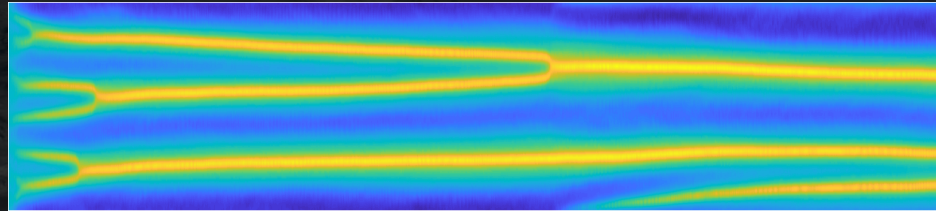
time

NONLINEAR (NL) MODEL – TYPES OF VARIABILITY

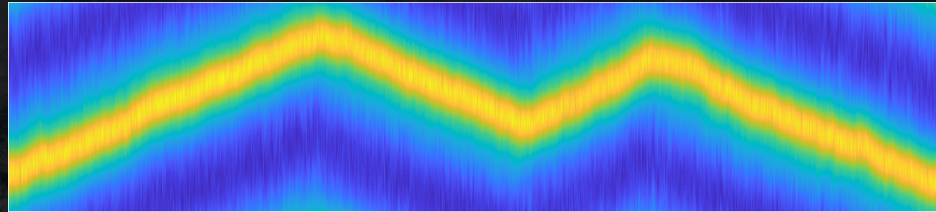
Randomly
wandering



Merging &
nucleating



Migrating



Result 1

New type of
variability
found: jets
migrate north
and south

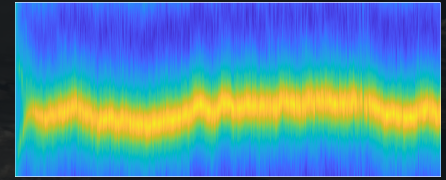
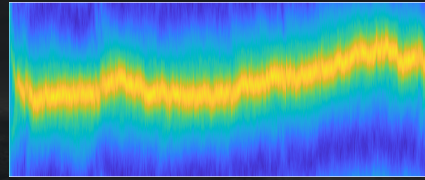
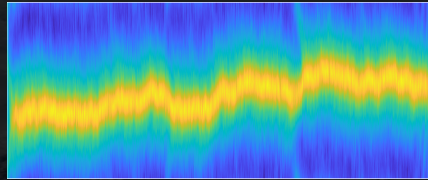
ALL MODELS – TYPES OF VARIABILITY

NL Model ($\Lambda = N$)

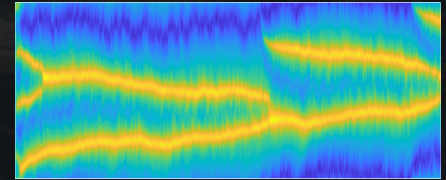
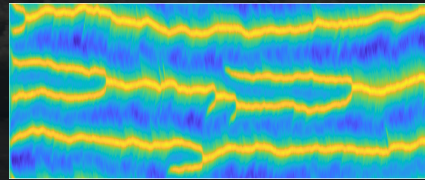
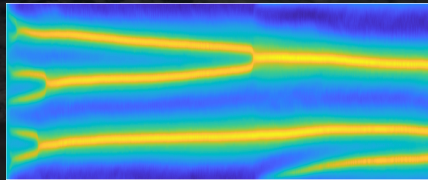
GQL Model ($\Lambda = 1$)

QL Model ($\Lambda = 0$)

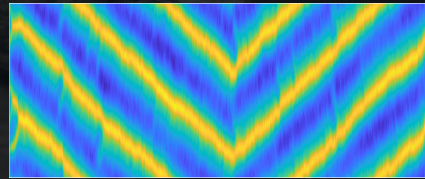
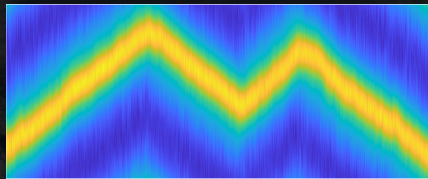
Randomly
wandering



Merging &
nucleating



Migrating



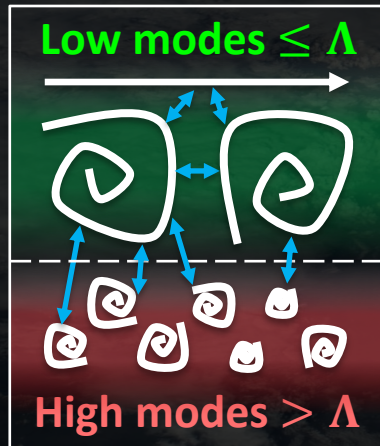
No clear
migration



A CLOSER LOOK AT ZONAL JET MIGRATION

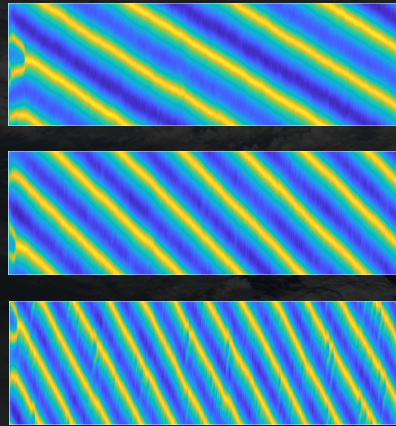
Question 1

Why do jets migrate only when $\Lambda \geq 1$?



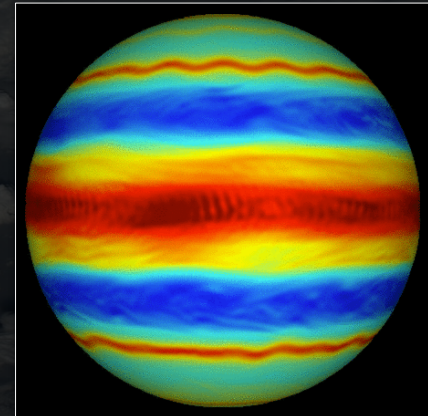
Question 2

Can we predict the speed of migration?



Question 3

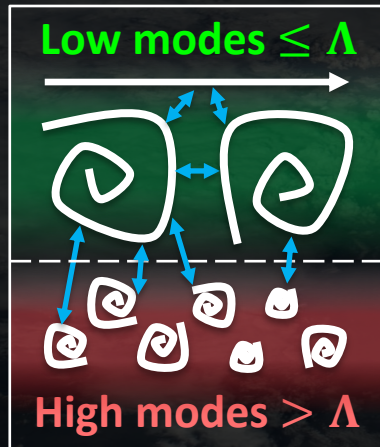
Do jets migrate in more complex systems?



A CLOSER LOOK AT ZONAL JET MIGRATION

Question 1

Why do jets migrate only when $\Lambda \geq 1$?



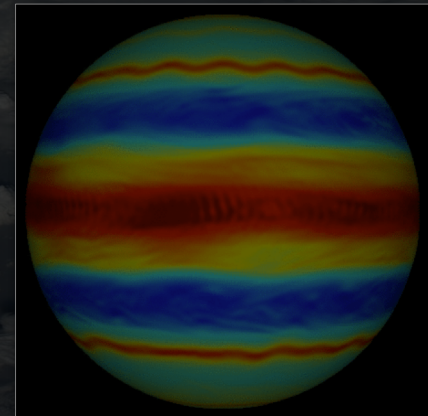
Question 2

Can we predict the speed of migration?



Question 3

Do jets migrate in more complex systems?



AN INTRODUCTION TO ZONONS

Zonons (Nonlinear waves)

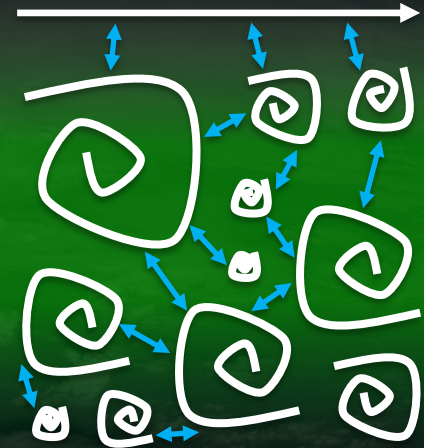
Coherent structures excited by Rossby waves with *same* k_x and *same* phase speed

Linear Rossby wave



Nonlinear zonons

NL Model ($\Lambda = N$)



Reference: Sukoriansky, Dikovskaya, Galperin (2008), PRL

AN INTRODUCTION TO ZONONS

Zonons (Nonlinear waves)

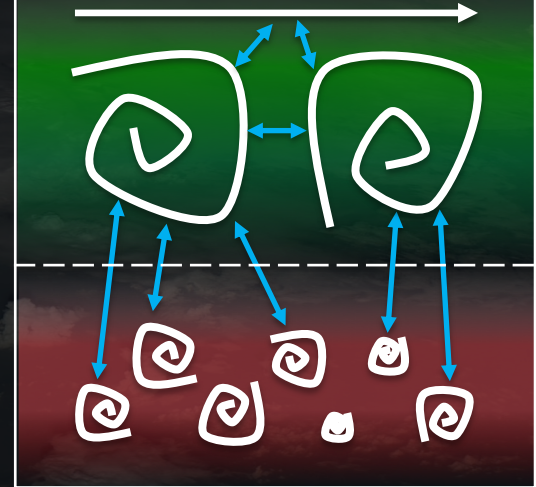
Coherent structures excited by Rossby waves with *same* k_x and *same* phase speed

Linear Rossby wave



Nonlinear zonons

GQL Model ($\Lambda = 1$)



Reference: Sukoriansky, Dikovskaya, Galperin (2008), PRL

AN INTRODUCTION TO ZONONS

Zonons (Nonlinear waves)

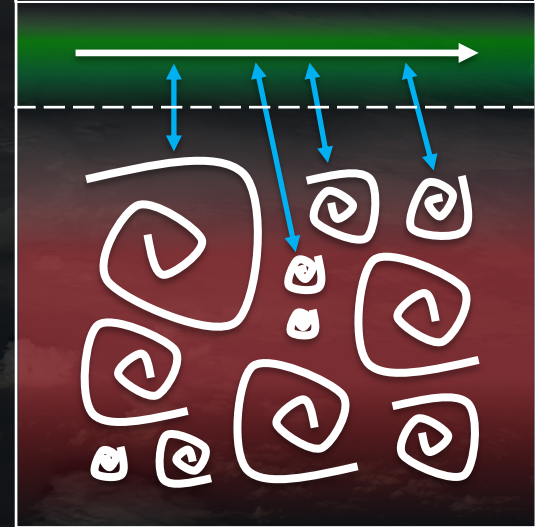
Coherent structures excited by Rossby waves with *same* k_x and *same* phase speed

Linear Rossby wave



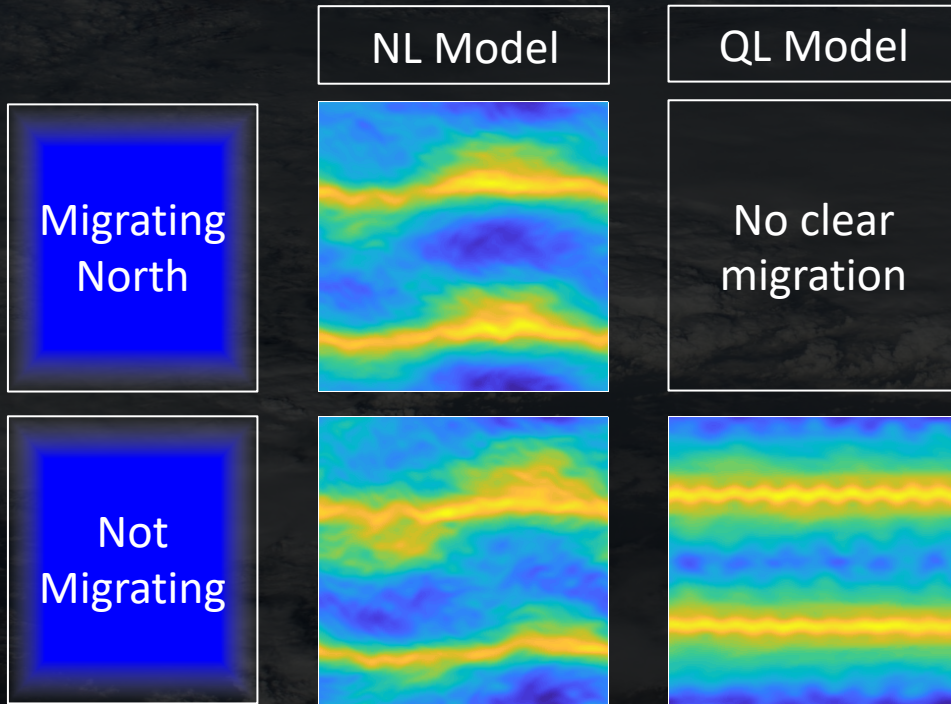
Nonlinear zonons

QL Model ($\Lambda = 0$)



Reference: Sukoriansky, Dikovskaya, Galperin (2008), PRL

Q1: WHY DO JETS MIGRATE ONLY WHEN $\Lambda \geq 1$?

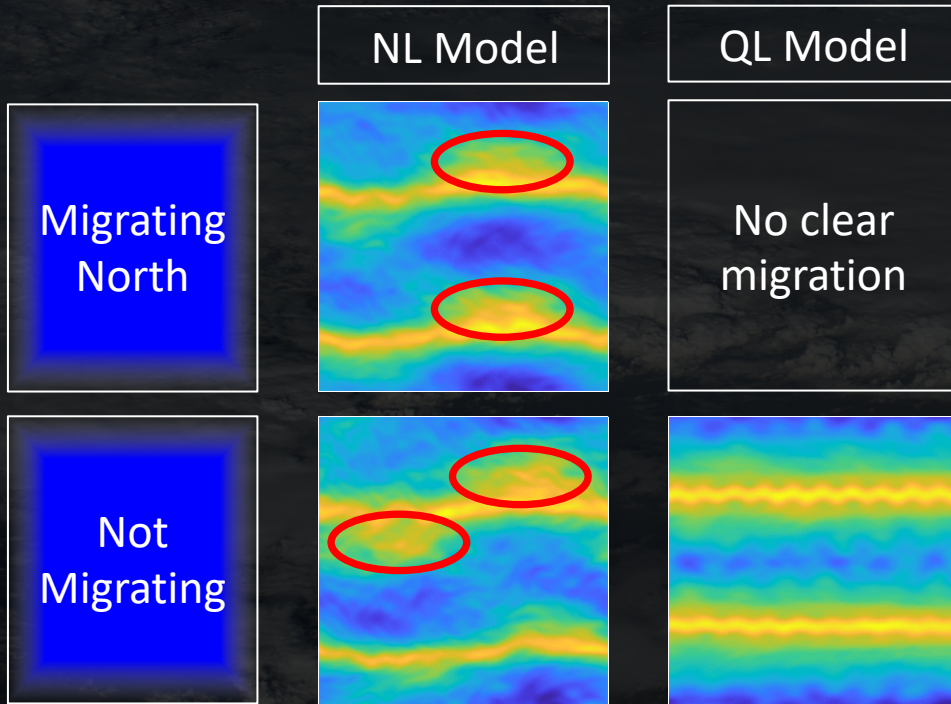


Result 2

Migration requires $\Lambda \geq 1$ when jets and zonons coexist


zonons
jet

Q1: WHY DO JETS MIGRATE ONLY WHEN $\Lambda \geq 1$?



Result 2

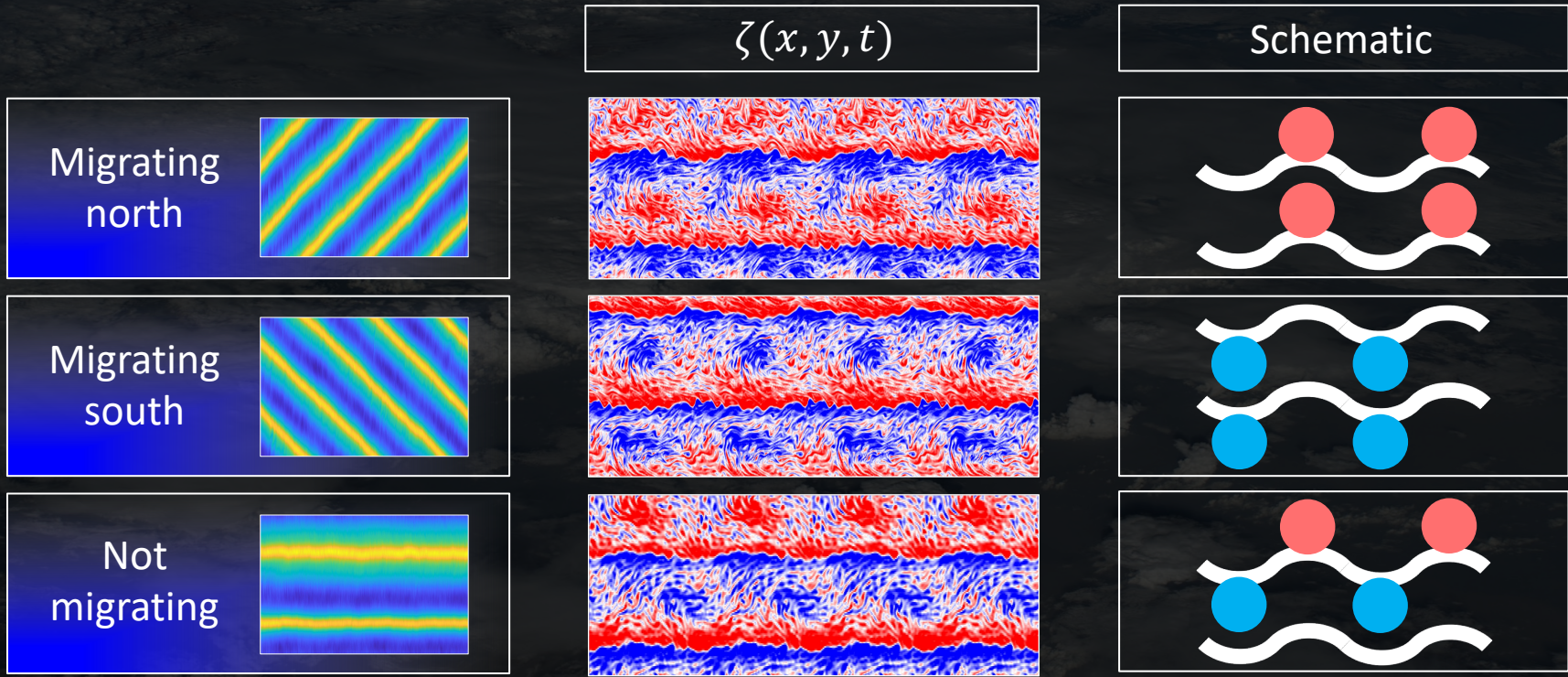
Migration requires $\Lambda \geq 1$ when jets and zonons coexist



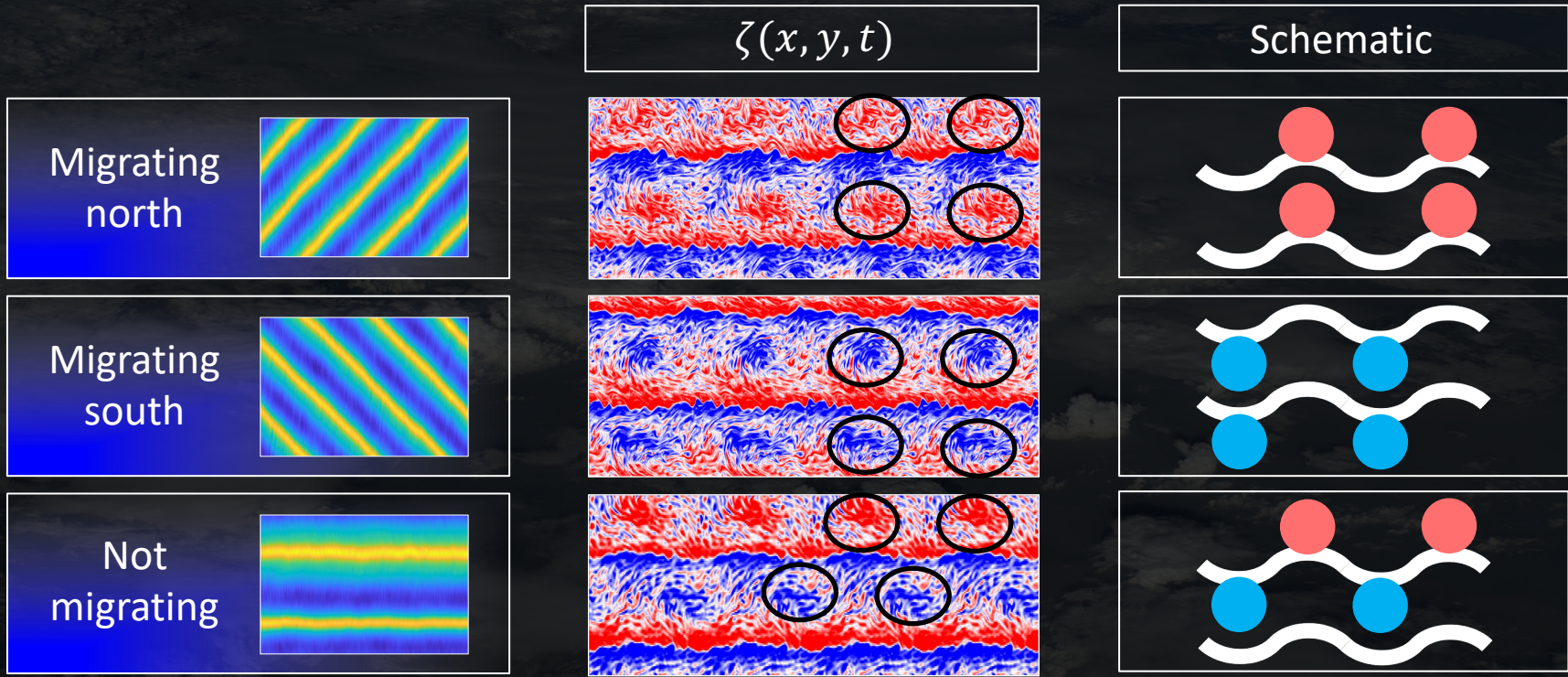
zonons

jet

Q1: WHY DO JETS MIGRATE ONLY WHEN $\Lambda \geq 1$?



Q1: WHY DO JETS MIGRATE ONLY WHEN $\Lambda \geq 1$?



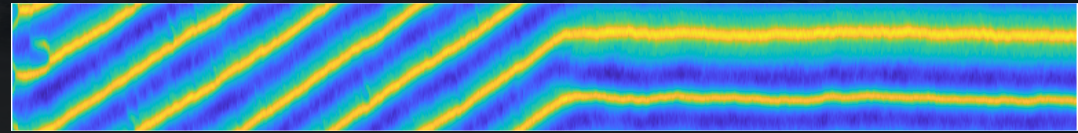
Q1: WHY DO JETS MIGRATE ONLY WHEN $\Lambda \geq 1$?

Zonal mean forcing

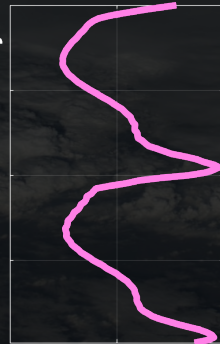
$$\frac{\partial \bar{u}}{\partial t} = \overline{v'\zeta'} - \mu \bar{u}$$

Reynolds
stress force

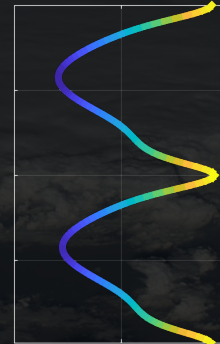
Zonal mean
flow



Latitude relative to jet

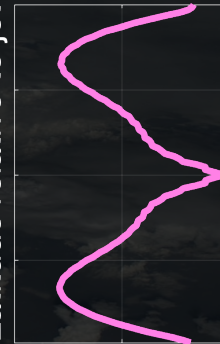


$\langle v'\zeta' \rangle$

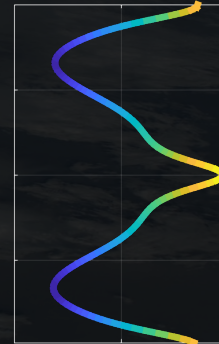


$\langle \bar{u} \rangle$

Latitude relative to jet



$\langle v'\zeta' \rangle$



$\langle \bar{u} \rangle$

Result 3

Migration requires an asymmetric eddy forcing and mean flow

A CLOSER LOOK AT ZONAL JET MIGRATION

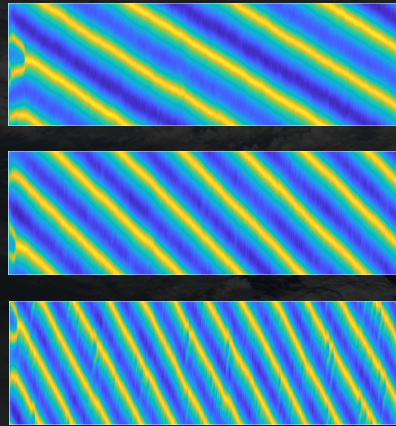
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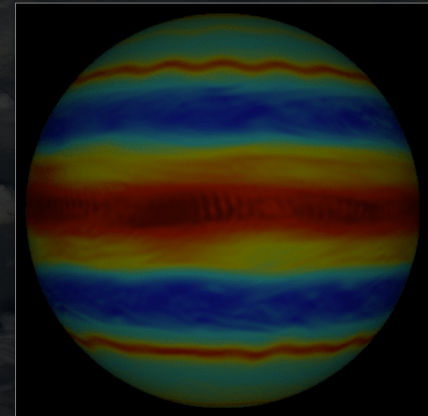
Question 2

Can we predict the speed of migration?

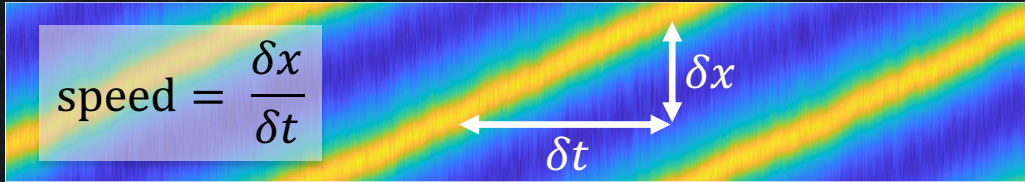


Question 3

Do jets migrate in more complex systems?

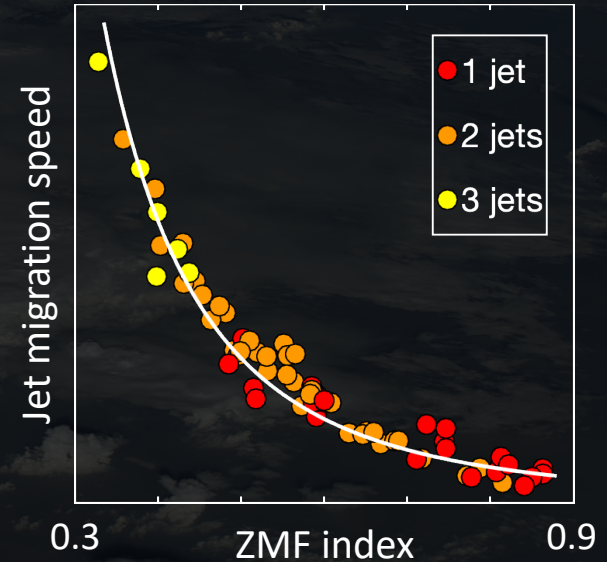
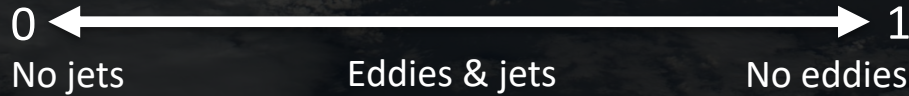


Q2: CAN WE PREDICT THE SPEED OF MIGRATION?



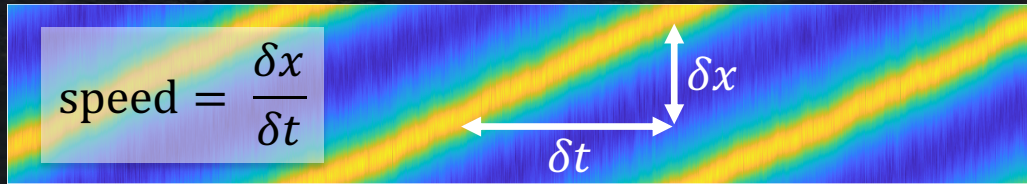
Zonal mean flow index (ZMF)

ZMF = Fraction of energy in mean flow



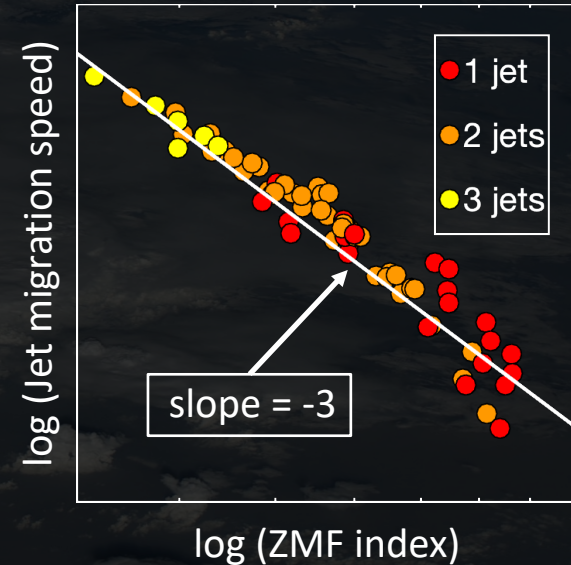
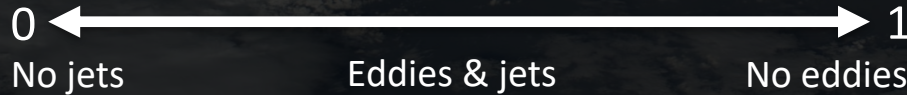
Result 4 Migration speed is given approximately by: $\text{speed} \propto (\text{ZMF})^{-3}$

Q2: CAN WE PREDICT THE SPEED OF MIGRATION?



Zonal mean flow index (ZMF)

ZMF = Fraction of energy in mean flow



Result 4 Migration speed is given approximately by: $\text{speed} \propto (\text{ZMF})^{-3}$

A CLOSER LOOK AT ZONAL JET MIGRATION

Question 1

Why do jets migrate only when $\Lambda \geq 1$?



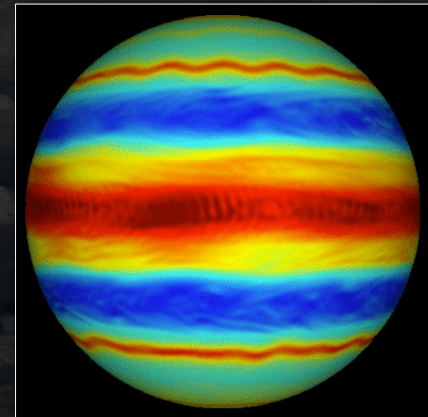
Question 2

Can we predict the speed of migration?



Question 3

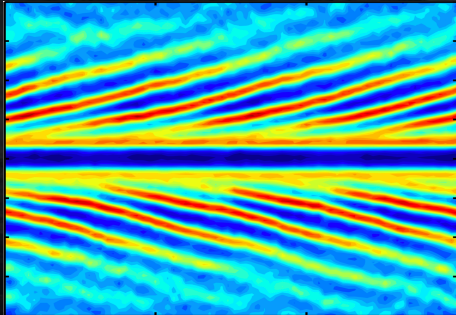
Do jets migrate in more complex systems?



Q3: DO JETS MIGRATE IN MORE COMPLEX SYSTEMS?

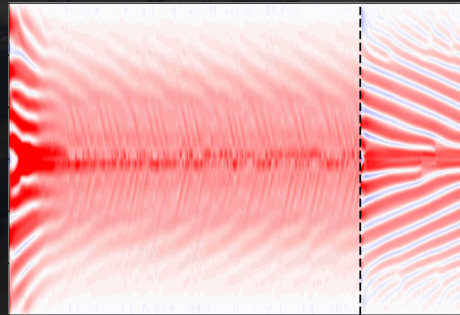
General circulation model:
Poleward drift

Kemke & Kaspi (2015),
JAMES



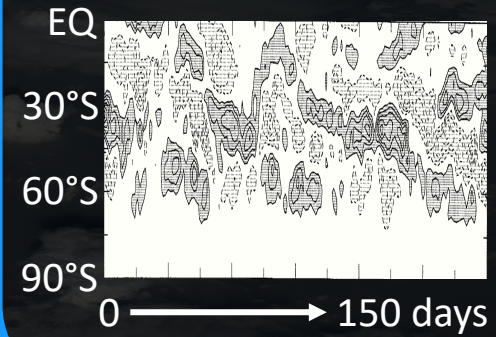
General circulation model:
Equatorward drift

Young, Read & Wang
(2019), Icarus



Atmosphere observations:
Poleward drift

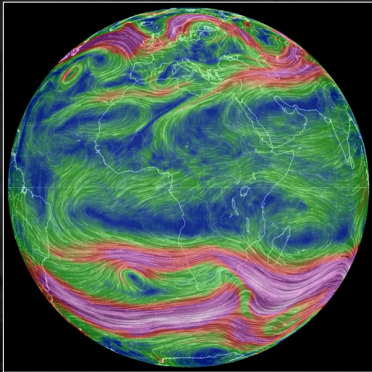
Feldstein (1998), JAS



CONCLUSIONS

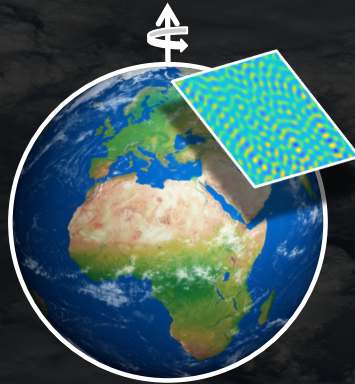
OBJECTIVE

Study of jet stream
variability



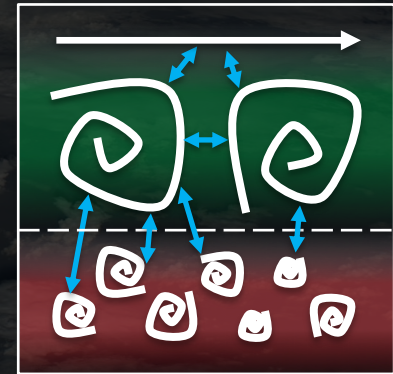
MODELS

Rotation + turbulence
+ friction = zonal jet



METHOD

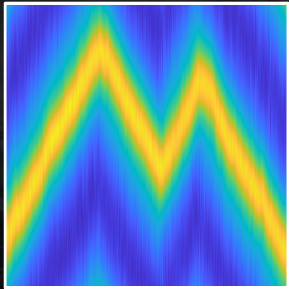
Generalized quasilinear
approximation



CONCLUSIONS

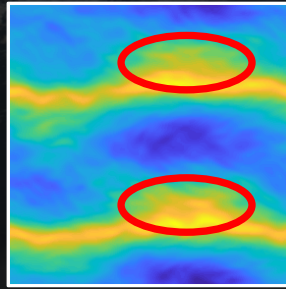
1

New type of jet
variability found:
migrating jets



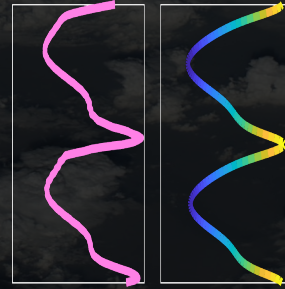
2

Migration requires
 $\Lambda \geq 1$ when jets
and zonons coexist



3

Migration requires
an asymmetric
eddy forcing



4

Migration speed is
approximately:
speed $\propto (\text{ZMF})^{-3}$

