Magnetohydrodynamic Turbulence in Stochastic Accretion Flows

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The talk was based on the following papers

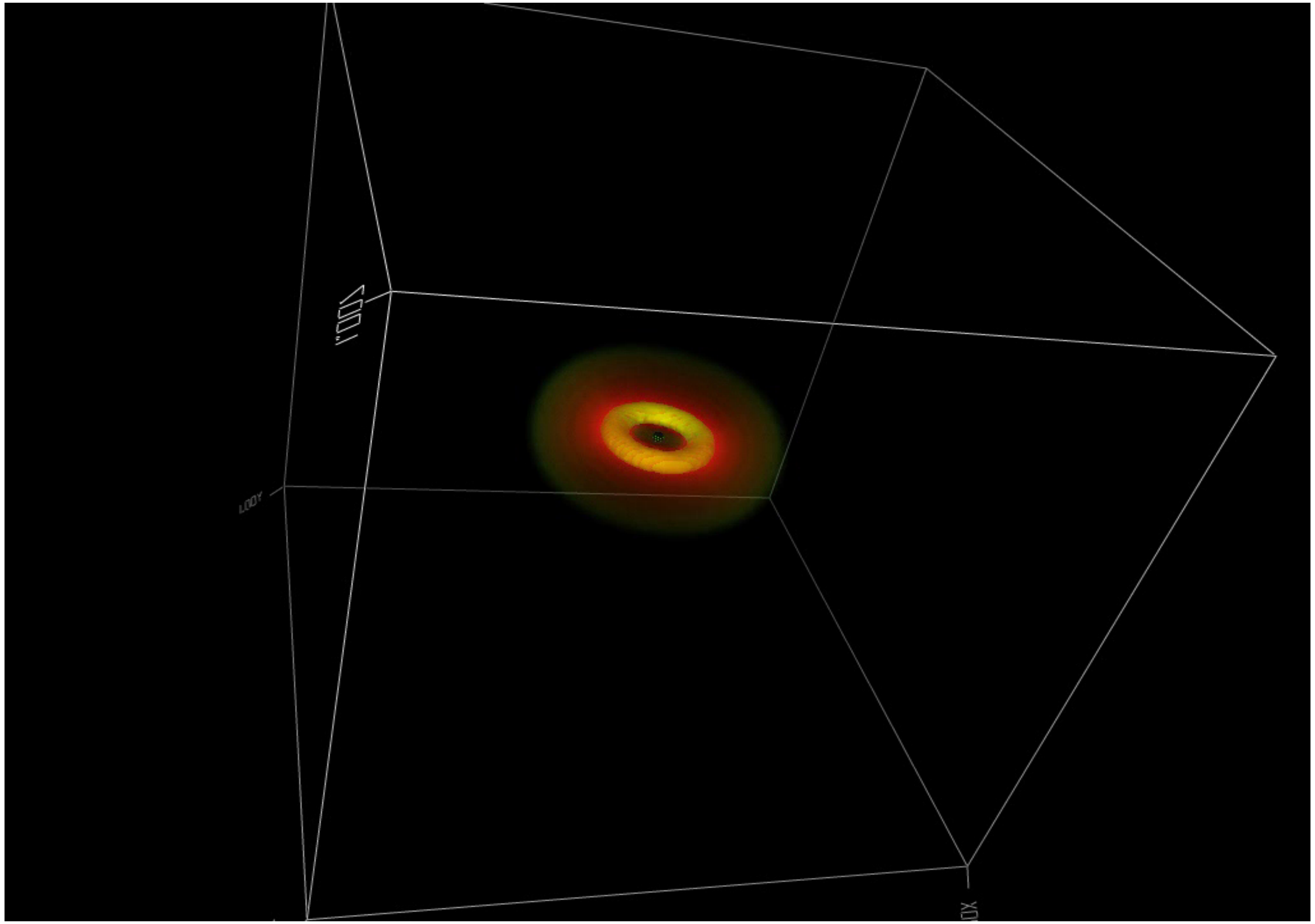
- S. K. Nath, AKC, *Cross-correlation-aided transport in stochastically driven accretion flows*, *Phys. Rev. E* ***90***, *063014, 2014*
- S. K. Nath, B. Mukhopadhyay, AKC, *Magnetohydrodynamic stability of stochastically driven accretion flows*, *Phys. Rev. E* ***88***, *013010, 2013*
- B. Mukhopadhyay, AKC, *Stochastically driven instability in rotating shear flows*, *J. Phys. A* **46**, 035501, 2013
- AKC, J. K. Bhattacharjee, *Wall-bounded turbulent shear flow: Analytic result for a universal amplitude*, *Phys. Rev E* **63**, 016306, 2000.

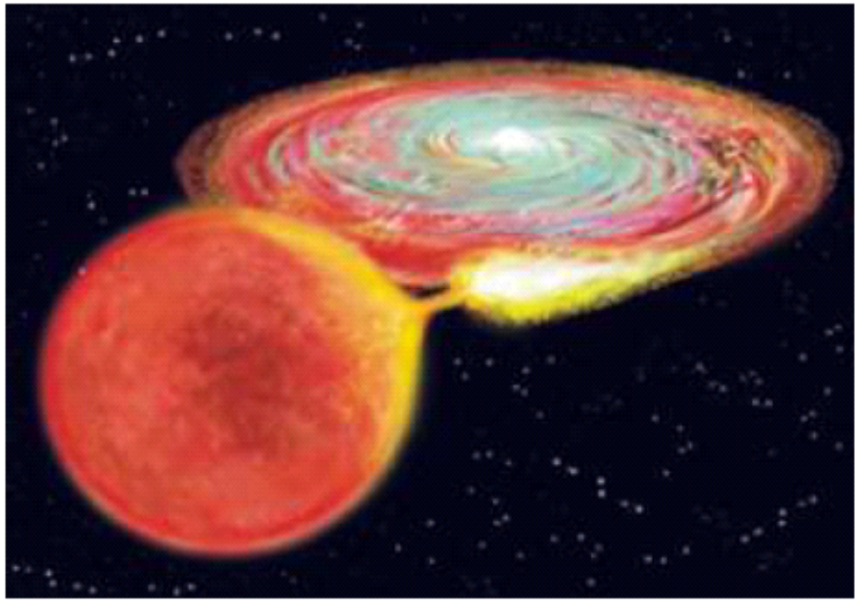
- B. Mukhopadhyay, N. Afshordi, R. Narayan, Bypass to turbulence in hydrodynamic accretion disks: An eigenvalue approach, *ApJ* **629**, 383, 2005

Followed up by (to be submitted)

- L. Godwin, S. C. Generalis, AKC, Stochastic stabilization of transient axisymmetric Taylor-Couette flow; Physical Review E.

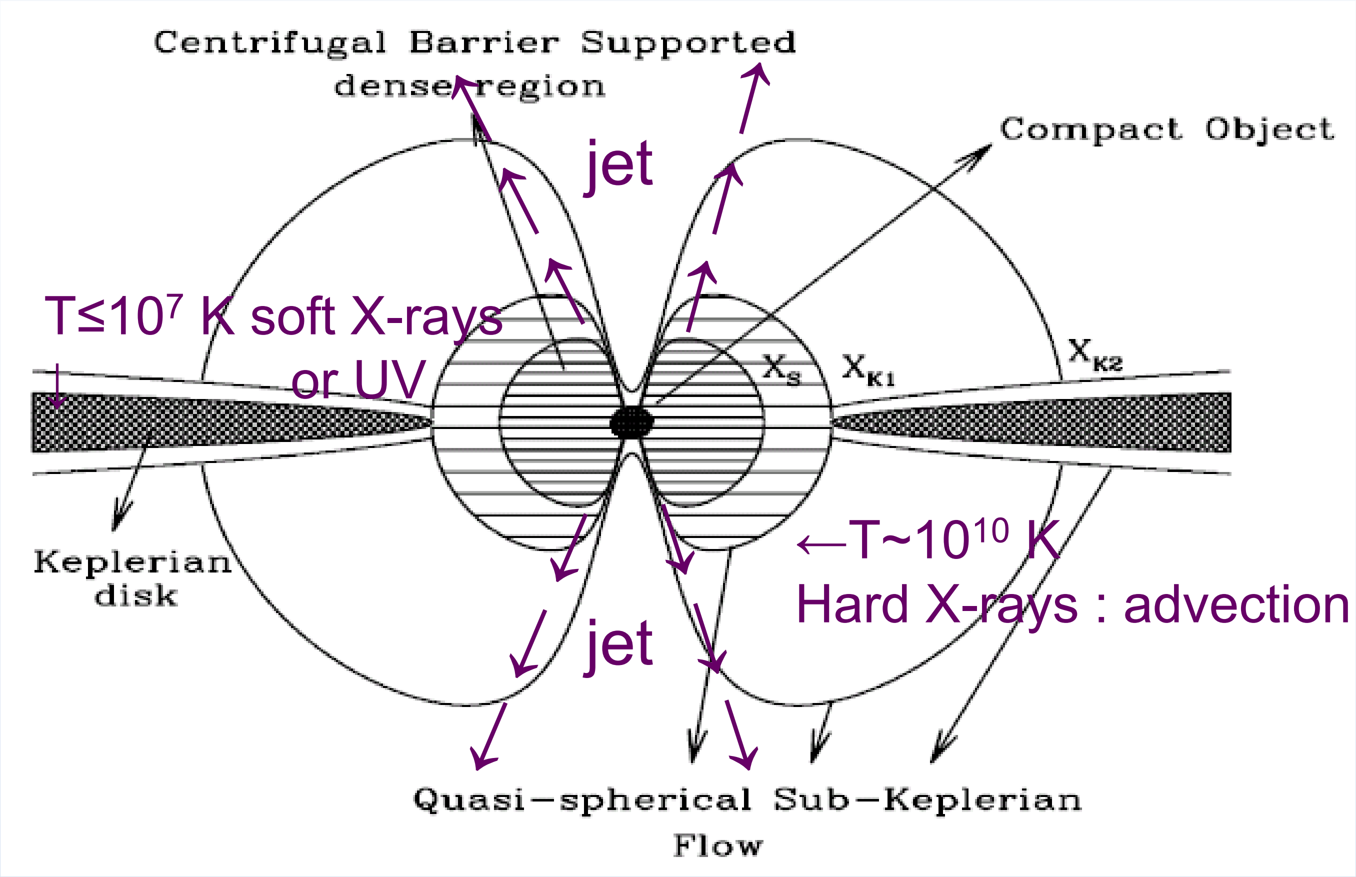
Accretion-Jet Formation





Accretion flows

A rotating mass (e.g. disc) in a closed orbit around a central gravitating body (e.g. black hole) can emit radiation when energy & angular momentum are extracted ➔ inward spiralling orbit



Radiation due to viscous dissipation ➔ Gravitational energy loss; **but Rayleigh stability criterion is still satisfied:** $\frac{d}{dR}(R^2\Omega) > 0$

Instability in an Accretion Disc: Viscous vs Thermal

- **Thermal instability:** Thermal time scale $T_h \sim \frac{\Sigma\, c_s^2}{D(R)}$, where Σ = disc surface density = ρH, H= disc thickness, c_s= sound speed, $D(R)$= local heat dissipation rate ~ $\nu\Sigma\Omega^2$

  ➔ $T_h \sim \left(\frac{H}{R}\right)^2 T_\nu \ll T_\nu$, where kinematic viscosity $\nu = \alpha c_s H$, $T_\nu = \frac{R^2}{\nu}$.

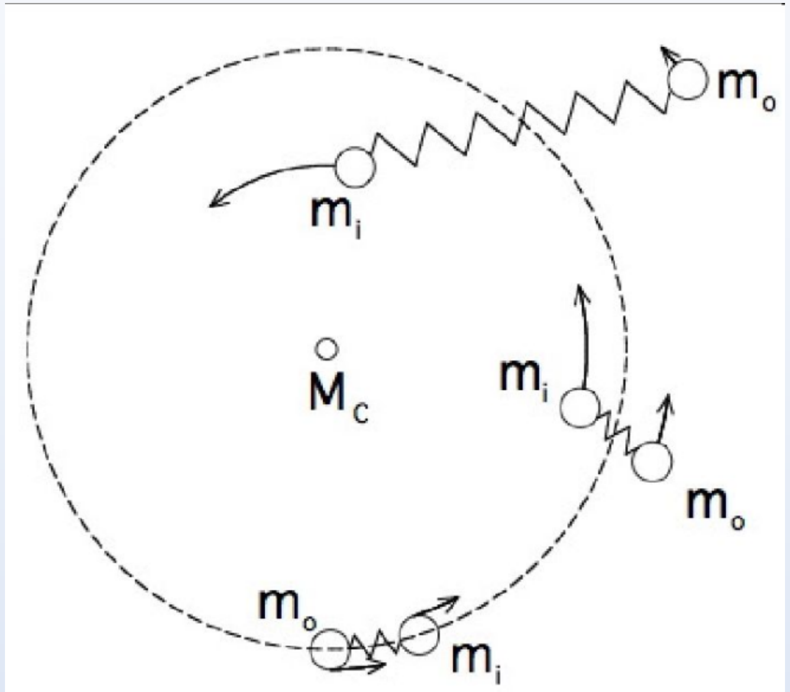
  Thus thermal instability grows faster than viscous instability

- **Viscous instability:** Criterion for viscous instability: $\frac{\partial}{\partial\Sigma}(\nu\Sigma) < 0$

If a disc is viscously unstable, the central locally under-dense region depletes faster than its edge ➔ accretes

- **Magnetic instability:** Due to shear across parallel fluid layers, driven by magnetic stress → magneto-rotational-instability. (MRI)

Magneto-Rotational-Instability (MRI)



Balbus-Hawley 1991: Magnetic fields generate angular momentum transport, e.g. a gas disc in the presence of a weak axial magnetic field. Two parallel fluid layers will behave as two mass points connected by a massless spring, the spring tension playing the role of the magnetic tension. In a Keplerian disk the inner fluid element would be orbiting faster, causing the spring to stretch: $\Omega \sim r^{-3/2}$

New stability criterion: $\frac{d}{d\,(\ln R)}(\Omega^2) > 0$

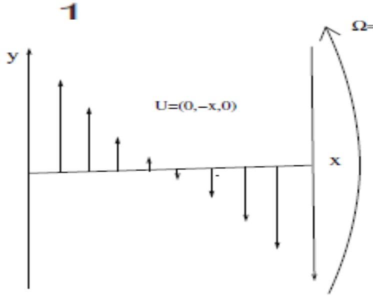
Mukhopadhyay-Afshordi-Narayan ApJ 2005: Fast transient growth in instability due to non-normal modes, arising out of Coriolis force. The energy grows by more than a factor of 100 for a Reynolds number R = 300 and more than a factor of 1000 for R = 1000 ➔ Turbulence. For a Keplerian disk, similar perturbations with vertical structure grow by no more than a factor of 4, explaining why the same simulations did not find turbulence in this system.

General set of MHD equations: Hot flows

$$\delta\dot{\mathbf{v}} = -\frac{1}{\rho}c_s^2\nabla\delta\rho + \nu\nabla^2\delta\mathbf{v} + 2\delta\mathbf{v}\times\mathbf{\Omega} + \quad \cdot\, \mathbf{q},$$

$\Omega \sim 1/r^q$,

$$\delta\dot{\rho} = -\rho\nabla\cdot\delta\mathbf{v}$$



In a local shearing box

Figure 1. Background unperturbed flow in the local comoving box. The size of arrows indicates the magnitude of the respective velocities.

$$\delta\dot{\mathbf{B}} = \nabla\times(\mathbf{v_0}\times\delta\mathbf{B} + \delta\mathbf{v}\times\mathbf{B}) + (\mathbf{v_0}\cdot\nabla)\delta\mathbf{B}, \qquad \nabla\cdot\delta\mathbf{B} = 0,$$

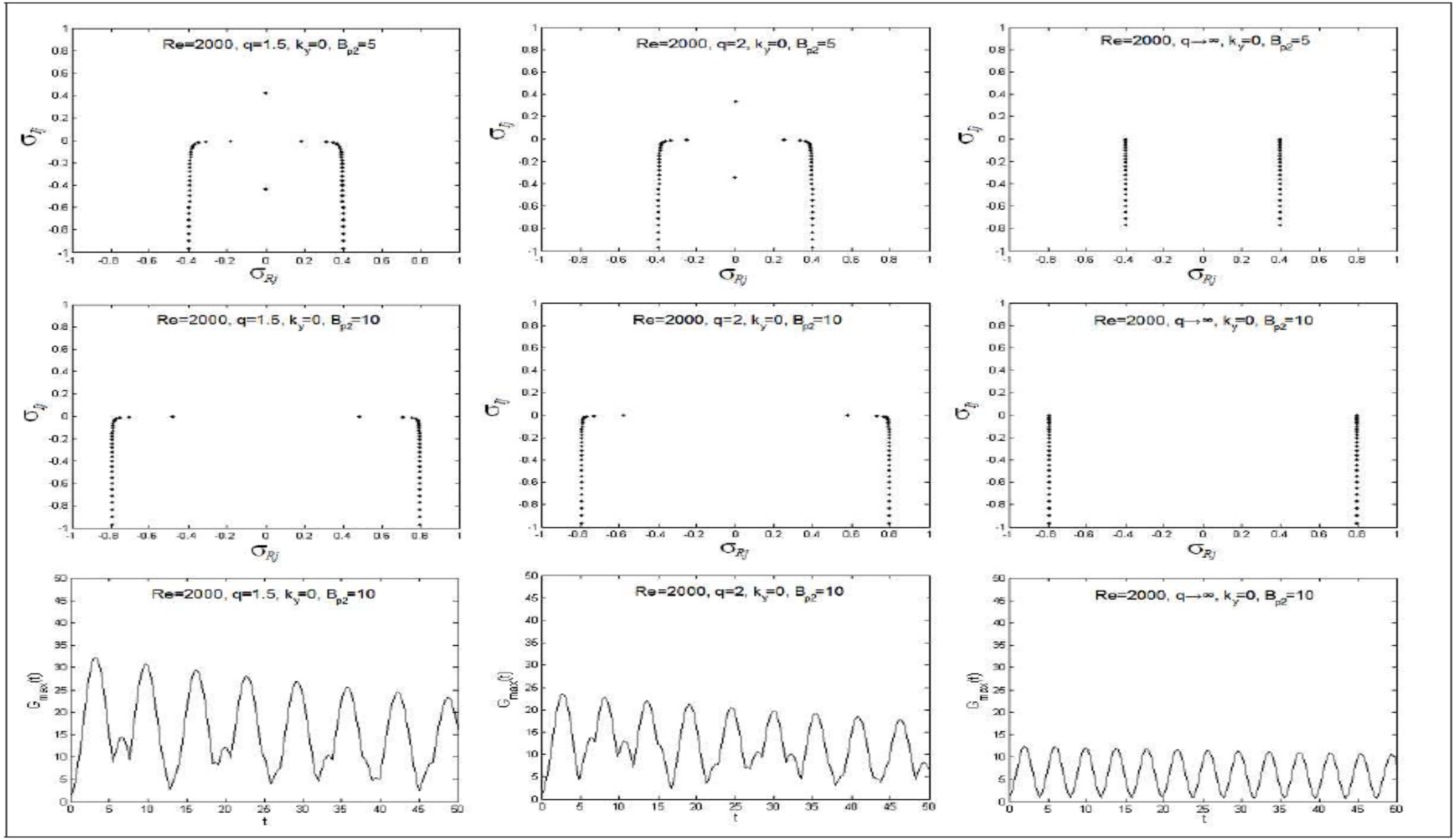
$$\delta\mathbf{v}, \delta\mathbf{B} \propto exp(i\mathbf{k}^L\cdot\mathbf{r}^L),$$

$$\mathbf{k} = (k_x, k_y, k_z) = (\mathbf{1} + \Omega t\mathbf{q})\cdot\mathbf{k}^L = (k_x^L + q\Omega t k_y^L, k_y^L, k_z^L),$$

$$\mathcal{E} \propto \left(\delta\mathbf{v}^2 + \frac{\delta\mathbf{B}^2}{4\pi\rho}\right).$$

Nath & Mukhopadhyay 2015

Complex plane (Argand diagram): Strong magnetic field effects



Bhatia & Mukhopadhyay 2016

FIG. 8: Eigenspectra in the presence of higher magnetic fields and corresponding energy growth, for $q = 1.5$ and 2 with $k_y = 0$.

Unanswered Questions from MRI Theory

1. Is there any pure hydrodynamic instability? NO! (Pringle, Ann. Rev. AA 1981).
2. What to do with **non-magnetic instability**?
3. How to incorporate the fact that **thermal instability > viscous instability**?
4. Can we accommodate the fact that even 'cold' **accretion stars are actually very hot - 10^3 C (cold X-ray star) to 10^6 C (hot X-ray star)**?

Our Hypothesis:

- Draw from De Dominicis-Martin (**PRA 19, 419, 1979**) Chattopadhyay-Bhattacharjee (**PRE 63, 0116306, 2000**) models of **stochastically forced Navier-Stokes flows in sheared coaxial cylinders** (total velocity $u = u_c + u_b$) that directly addresses points 2-4 above and laterally 1.
- **OUR MODEL**: **stochastically forced Orr-Sommerfeld and Squire equations** in presence of Coriolis force

Forced Orr-Sommerfeld Model: Magnetic vs Stochastic: Chattopadhyay-Mukhopadhyay Model

Plane shear: (0,-x,0); angular velocity: $\Omega \sim r^{-q}$; vorticity $\zeta = \nabla \times \mathbf{u}$ ➔ Linearized Hydrodynamic Model:

$$\left(\frac{\partial}{\partial t} - x\frac{\partial}{\partial y}\right)\nabla^2 u + \frac{2}{q}\left(\frac{\partial \zeta}{\partial z}\right) - \frac{1}{4\pi}(\overrightarrow{B_p}.\overrightarrow{\nabla})\nabla^2 B_x = \frac{1}{Re}\nabla^4 u,$$

$$\left(\frac{\partial}{\partial t} - x\frac{\partial}{\partial y}\right)\zeta + \left(1 - \frac{2}{q}\right)\left(\frac{\partial u}{\partial z}\right) - \frac{1}{4\pi}(\overrightarrow{B_p}.\overrightarrow{\nabla})\zeta_B = \frac{1}{Re}\nabla^2 \zeta,$$

Magnetized Non-normal model

$$\left(\frac{\partial}{\partial t} - x\frac{\partial}{\partial y}\right)\nabla^2 u + \frac{2}{q}\frac{\partial \zeta}{\partial z} = \frac{1}{Re}\nabla^4 u + \eta_1(x,t),$$

$$\left(\frac{\partial}{\partial t} - x\frac{\partial}{\partial y}\right)\zeta + \frac{\partial u}{\partial z}\left(1 - \frac{2}{q}\right) = \frac{1}{Re}\nabla^2 \zeta + \eta_2(x,t),$$

Stochastic Model

(magnetic fluctuations → noise)

$$< \eta_i(\mathbf{x},t)\eta_j(\mathbf{x}',t') > = D_i(|\mathbf{x}-\mathbf{x}'|)\delta(t-t')$$

$D_i(k) \sim k^{d-\alpha}, \alpha > 0$ ➔ Vertex correction

Noise Strength

Correlation Functions

- Fourier transforms: $\phi(\mathbf{x},t) = \int \phi_k \, e^{i(\boldsymbol{k}.\mathbf{x}-\omega t)} d^3k \, d\omega$, where $\phi = u, \zeta, \eta_i$
- Temporal Correlation Functions:
  1. $C_u(\tau) =< [u(\mathbf{x}, t+\tau) - u(\mathbf{x},t)]^2 > = \int < u_{\mathbf{k},\omega} u_{-\mathbf{k},-\omega} > e^{-i\omega\tau} \, d^3k \, d\omega$
  2. $C_\zeta(\tau) =< [\zeta(\mathbf{x}, t+\tau) - \zeta(\mathbf{x},t)]^2 > = \int < \zeta_{\mathbf{k},\omega} \zeta_{-\mathbf{k},-\omega} > e^{-i\omega\tau} \, d^3k \, d\omega$

$$\boldsymbol{C_u(\tau) = 4\pi \int_{k_0}^{k_m} dk \, k^2 \, RES_{C_u}(q, R_e, \tau);}$$

$$RES_{C_u} = \frac{2\sqrt{2}\tau \, \pi R_e \, [(bm-an)\sin(a)\cosh(b) + (am+bn)\cos(a)\sinh(b)]}{\sqrt{[3(2-q)q \, k^6 (a^2+b^2)}},$$

q=3/2 (Keplerian disc), 2 (constant angular momentum), 1 (flat rotation), 0 (solid body rotation)

$$a = \sqrt{[2x_1 + \sqrt{4x_1^2 + y_1^2}}, b = \sqrt{\left[x_1 + \sqrt{4x_1^2 + y_1^2}\right]},$$

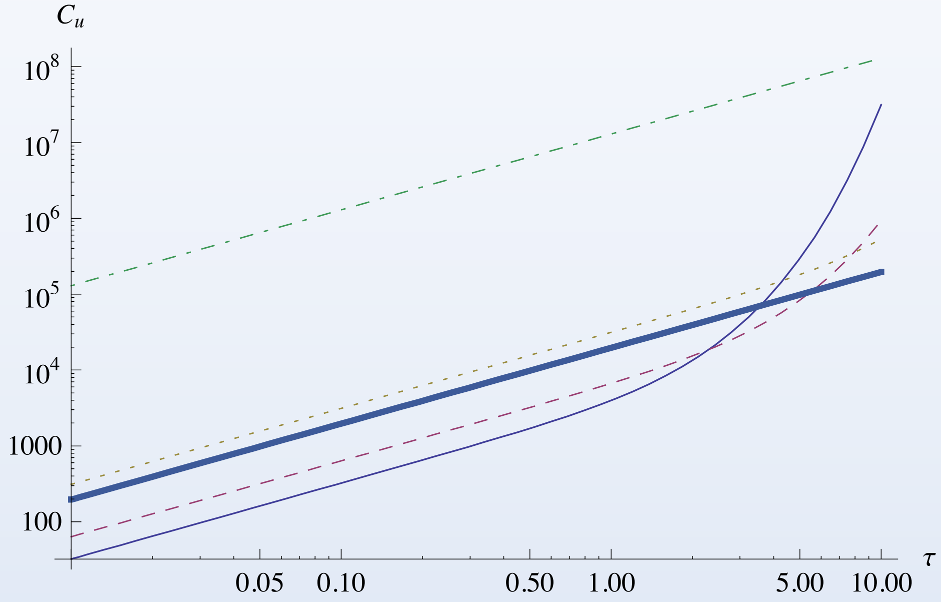
$$x_1 = -[\frac{k^4}{R_e^2} + \frac{2}{3}(\frac{2-q}{q^2})]\tau^2, y_1 = -[\frac{2}{3}\sqrt{(6(2-q)}(\frac{k^2}{q \, R_e})]\tau^2; \text{m} = 2k^2(2-q), n = \sqrt{(6(2-q)}\frac{qk^2}{R_e}$$

- Spatial Correlation Functions:

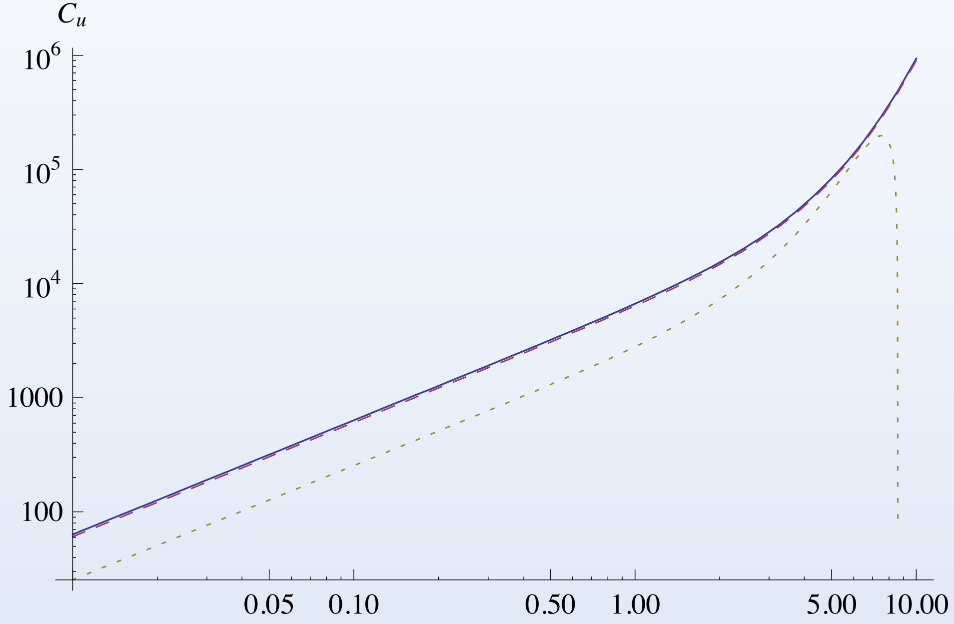
$$S_u(r) =< [u(\mathbf{x}+\mathbf{r}, t) - u(\mathbf{x},t)]^2 > = \int < u_{\mathbf{k},\omega} u_{-\mathbf{k},-\omega} > e^{i\mathbf{k}.r} \, d^3k \, d\omega$$

$$S_\zeta(r) =< [\zeta(\mathbf{x}+\mathbf{r}, t) - \zeta(\mathbf{x},t)]^2 > = \int < \zeta_{\mathbf{k},\omega} \zeta_{-\mathbf{k},-\omega} > e^{i\mathbf{k}.r} \, d^3k \, d\omega$$

Temporal Correlation vs Time Difference
(Shows strong and fast growing instability)



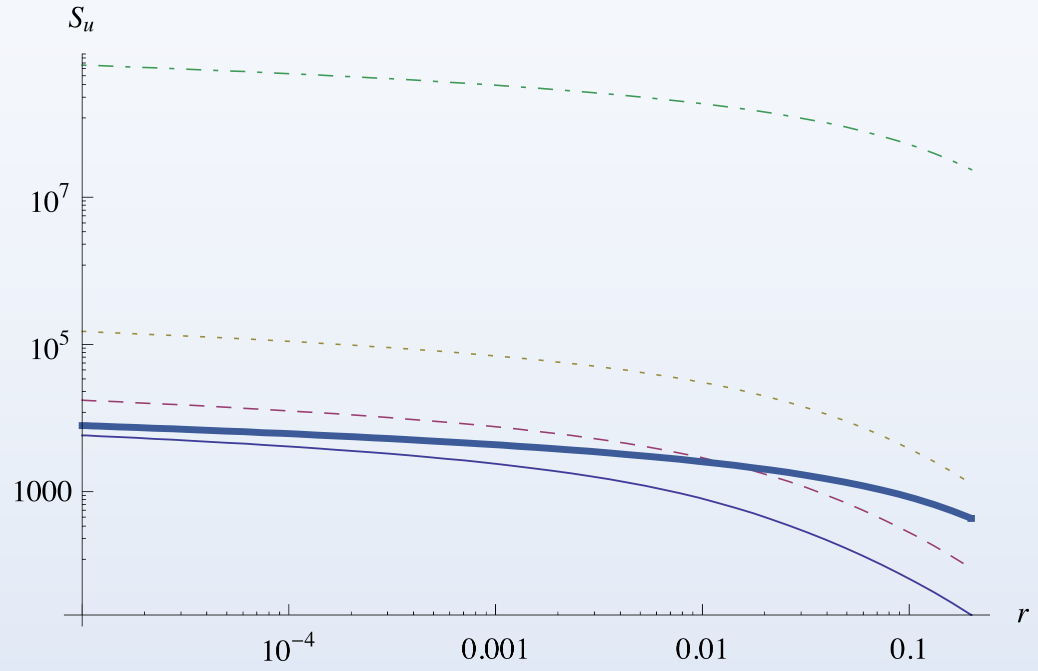
$\mathbf{R_e = 10,000;}$
q=1(solid), 1.5 (dashed), 1.9 (dotted), ~2 (dot-dashed)



$\boldsymbol{q = 1.5;}$
R_e = 10,000 (solid), 1000 (dashed), 100 (dotted)

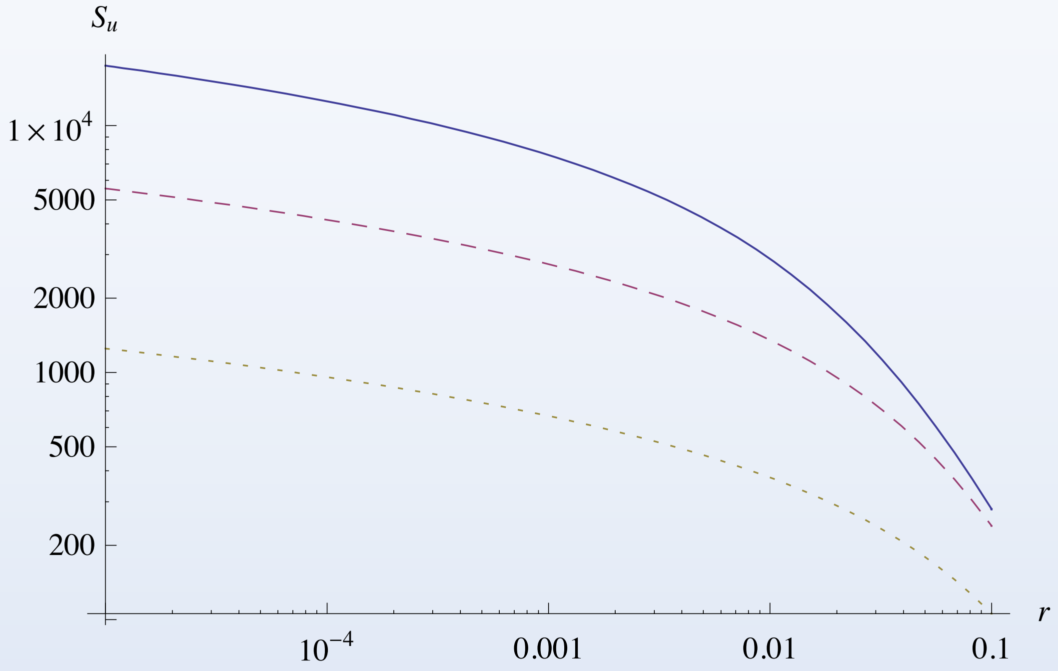
Spatial Correlation vs Length Scale

(Finite length suppression of spatial instability)



$\mathbf{R_e = 10,000;}$

q=1(solid), 1.5 (dashed), 1.9 (dotted), ~2 (dot-dashed)



$\boldsymbol{q = 1.5;}$

$\mathrm{R_e}$ = 10,000 (solid), 1000 (dashed), 100 (dotted)

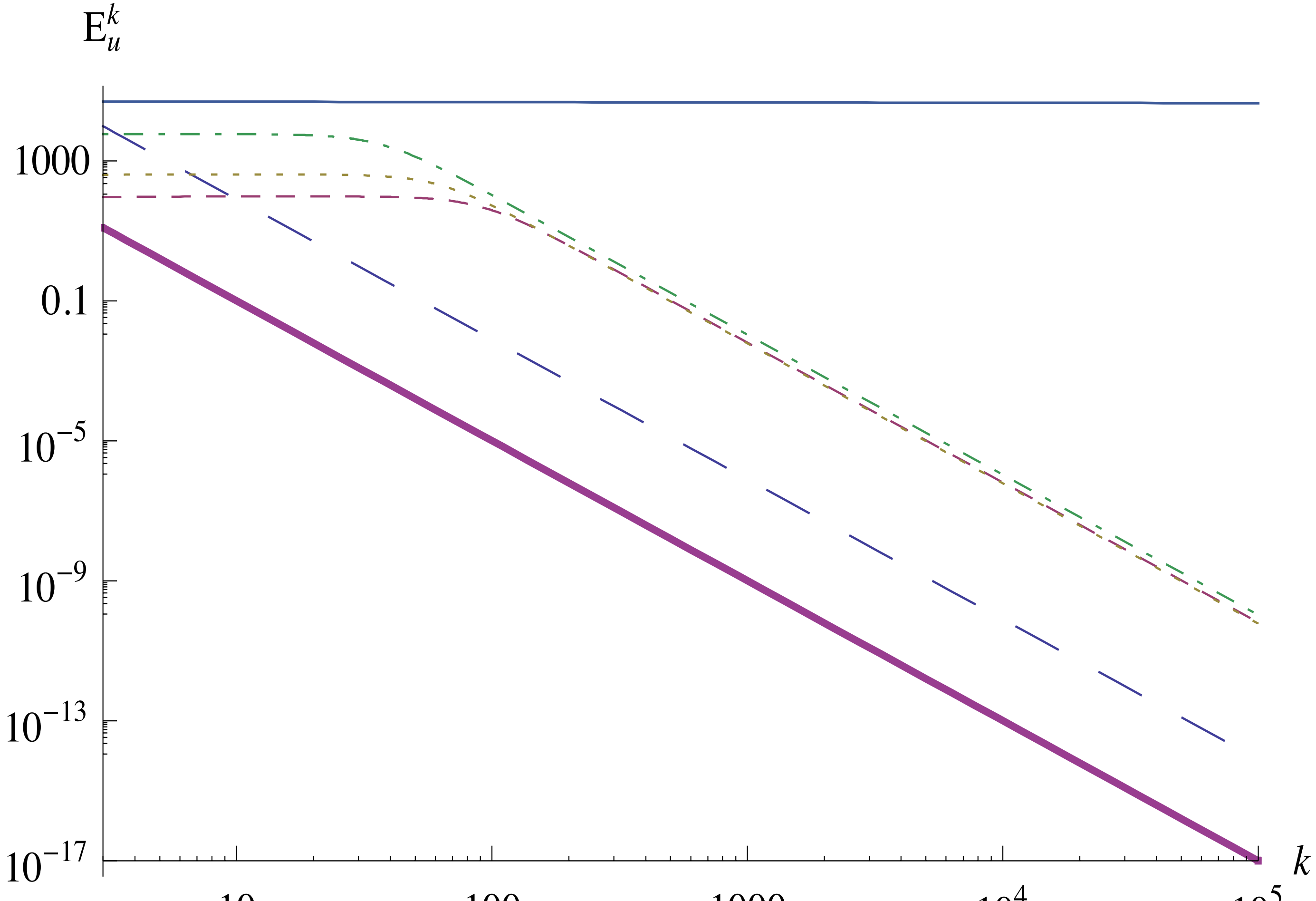
Energy Instability

–

INTERMITTENCY?

$\eta \sim 0.01 - 0.1$

$$\mathbf{E_u^k} = \frac{1}{8}\left[\mathbf{u_{rms}^2} + \frac{1}{\mathbf{k^2}}\left(\frac{\partial u}{\partial \mathbf{x}}\right)_{rms}^2 + \zeta_{rms}^2\right]$$



MAGNETIZED & Stochastically Forced Orr-Sommerfeld Model: Chattopadhyay-Nath Model

$$\left(\frac{\partial}{\partial t} - x\frac{\partial}{\partial y}\right)\nabla^2 u + \frac{2}{q}\frac{\partial \zeta}{\partial z} - \frac{1}{4\pi}\left(B_1\frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\nabla^2 B_x = \frac{1}{R_e}\nabla^4 u + \eta_1(\mathbf{x}, t)$$

$$\left(\frac{\partial}{\partial t} - x\frac{\partial}{\partial y}\right)\zeta + \frac{\partial u}{\partial z} - \frac{2}{q}\frac{\partial u}{\partial z} - \frac{1}{4\pi}\left(B_1\frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\zeta_B = \frac{1}{R_e}\nabla^4 \zeta + \eta_2(\mathbf{x}, t)$$

$$\left(\frac{\partial}{\partial t} - x\frac{\partial}{\partial y}\right)B_x - B_1\frac{\partial u}{\partial y} - \frac{\partial u}{\partial z} = \frac{1}{R_m}\nabla^4 B_x + \eta_3(\mathbf{x}, t)$$

$$\left(\frac{\partial}{\partial t} - x\frac{\partial}{\partial y}\right)\zeta_B - \frac{\partial \zeta}{\partial z} - B_1\frac{\partial \zeta}{\partial y} - \frac{\partial B_x}{\partial z} = \frac{1}{R_m}\nabla^4 \zeta_B + \eta_4(\mathbf{x}, t)$$

$$< \eta_i(\mathbf{x}, t)\eta_j(\mathbf{x}', t') > = D_i(|\mathbf{x} - \mathbf{x}'|)\delta(t - t')$$

Noise Strength

$D_i(k) \sim k^{d-\alpha}, \alpha > 0$ ➔ Vertex correction

SpatioTemporal Correlation vs Space/Time Difference

TEMPORAL CORRELATION

$\mathbf{R_e = 10,000;}$

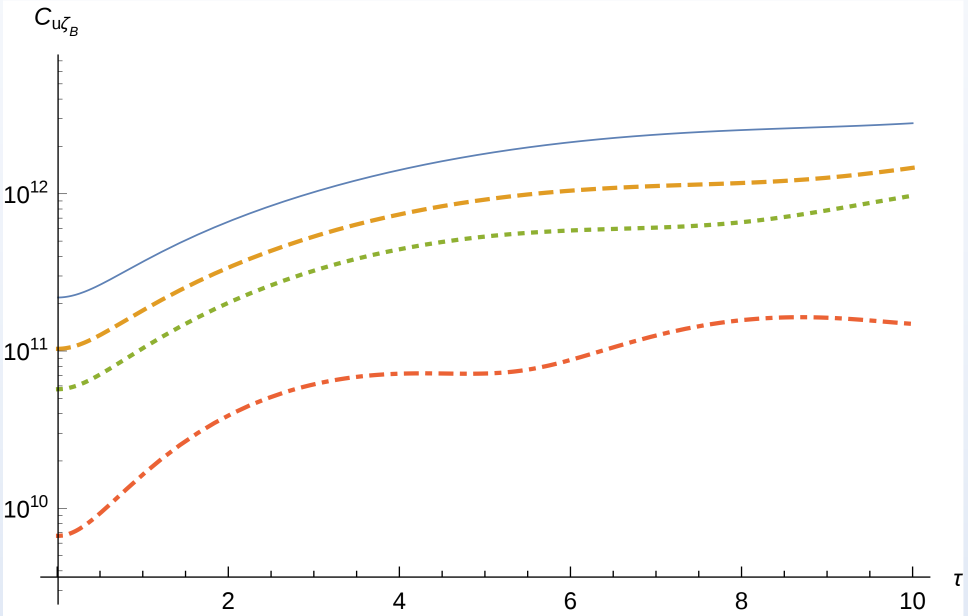
q=1.5 (solid), 1.7 (dashed), 1.9 (dotted), ~2 (dot-dashed)

SPATIAL CORRELATION

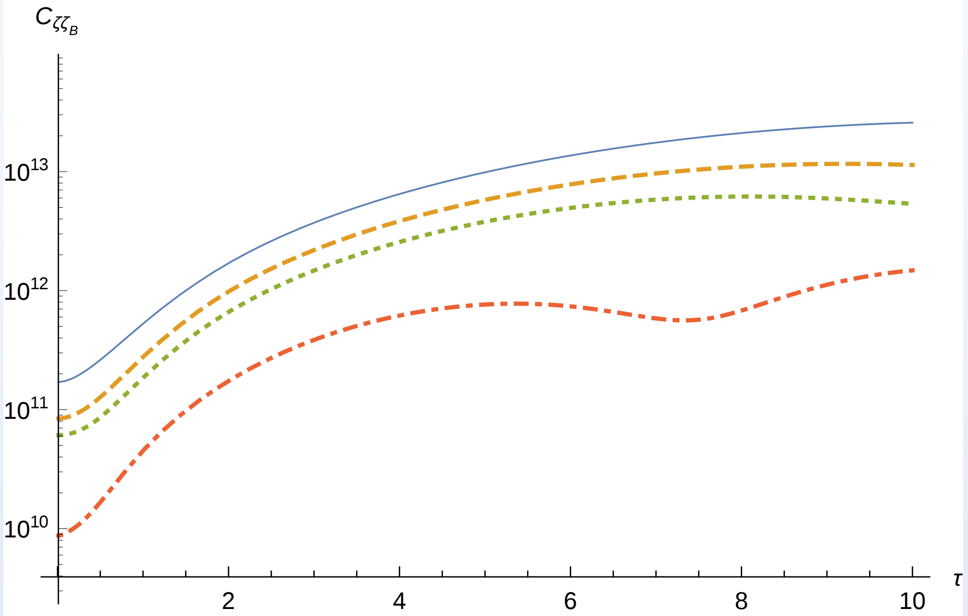
$\mathbf{R_e = 10,000;}$

q=1.5 (solid), 1.7 (dashed), 1.9 (dotted), ~2 (dot-dashed)

SpatioTemporal Cross-Correlations vs Space/Time Difference

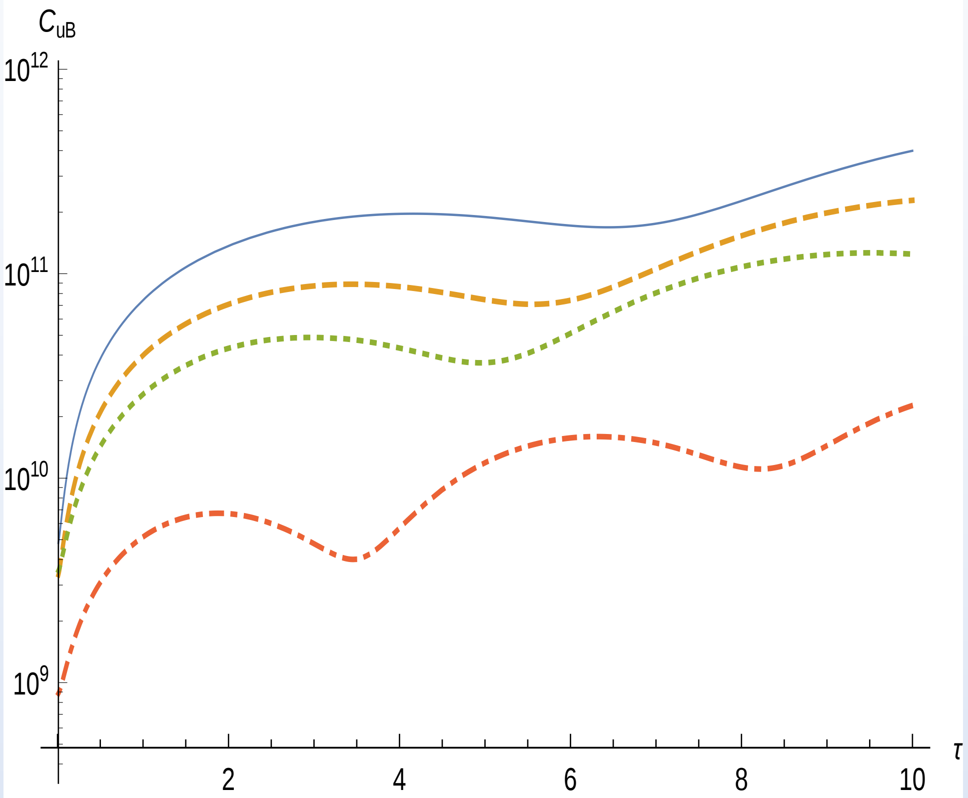


Velocity-Vorticity cross-correlation
q=1.5 (solid), 1.7 (dashed), 1.9 (dotted), ~2 (dot-dashed)

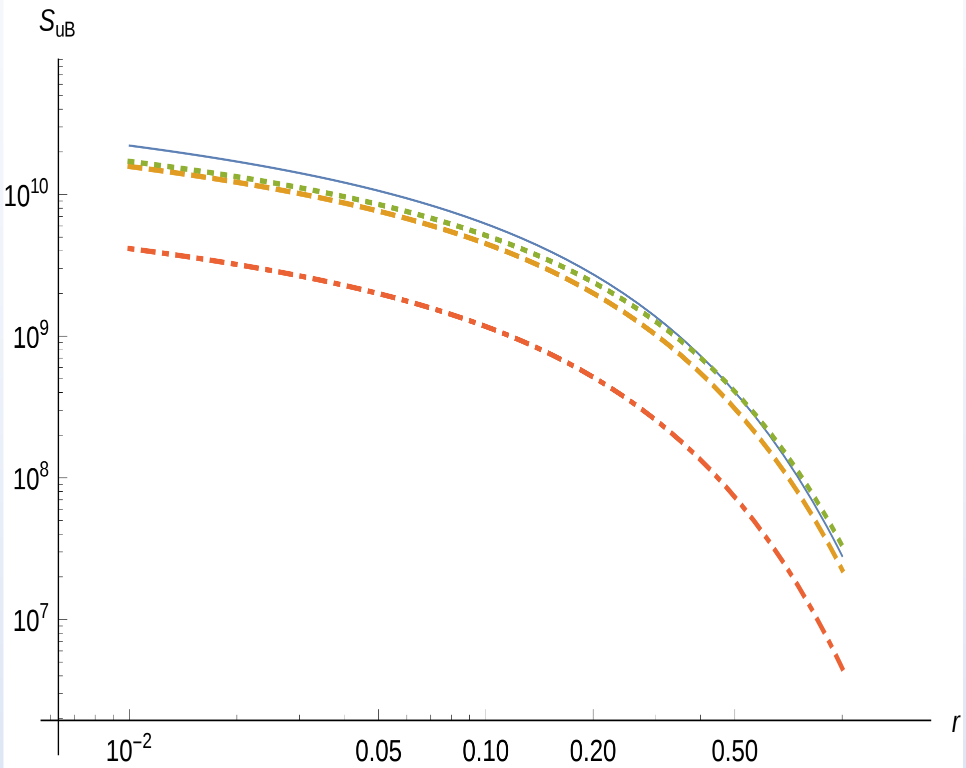


Vorticity-magnetic vorticity cross-correlation
q=1.5 (solid), 1.7 (dashed), 1.9 (dotted), ~2 (dot-dashed)

SpatioTemporal Cross-Correlations vs Space/Time Difference



Velocity-Magnetic field TEMPORAL cross-correlation
q=1.5 (solid), 1.7 (dashed), 1.9 (dotted), ~2 (dot-dashed)



Vorticity-magnetic vorticity SPATIAL cross-correlation
q=1.5 (solid), 1.7 (dashed), 1.9 (dotted), ~2 (dot-dashed)

Summary and Conclusions

- Origin of instability and then plausible turbulence in accretion discs and similar laboratory shear flows is generally a big question.

- Magnetorotational Instability (MRI) is a good mechanism for magnetic accretion discs, but it is limited to weak field regime and not applicable to laboratory flows. But that does not explain instability in non-magnetic accretion flows.

- **Stochastic Forcing can bridge the magnetic & non-magnetic accretion ends.**

- **Cross-correlations lead to time symmetry violation that 'adds on' to the Coriolis force which in turn leads to suppression of Alfven waves at finite time scales.**