Coherent structures in Quantum Turbulence

(Waves are present...but not discussed)

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Classical (viscous) turbulence

- In a 3D classical turbulent flow, large scale eddies break up into smaller eddies, these into smaller ones and so on...(Richardson Cascade)

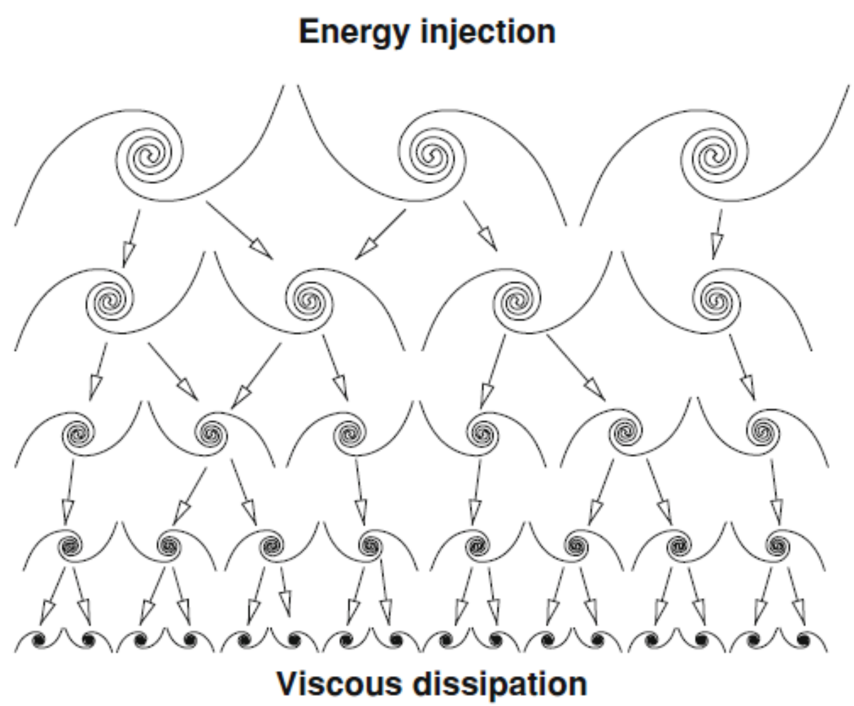
$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v}$$

$$v = \sin(x) \Rightarrow v\frac{\partial v}{\partial x} \sim \sin(2x)$$

- If there is a large inertial range between the forcing and dissipation scale (i.e. high Re) then the flow of energy through scales is characterized by a constant energy flux .

- Dimensional analysis leads to a power-law scaling for the energy spectrum,

$$E(k) = C\epsilon^{2/3}k^{-5/3}$$



Classical Vorticity

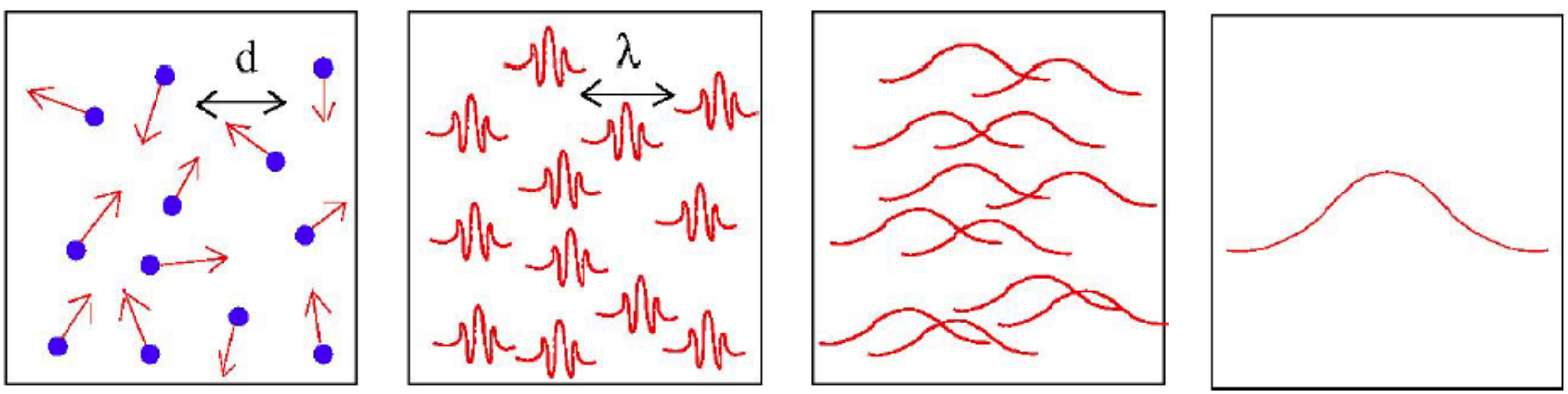
$$\boldsymbol{\omega} = \nabla \times \mathbf{u}$$

Quantum Fluids

- Atom with momentum $p = mv$ has wavelength $\lambda = h/p$
- Average kinetic energy $mv^2/2 \approx k_B T$
- Wavelength increases with decreasing T:

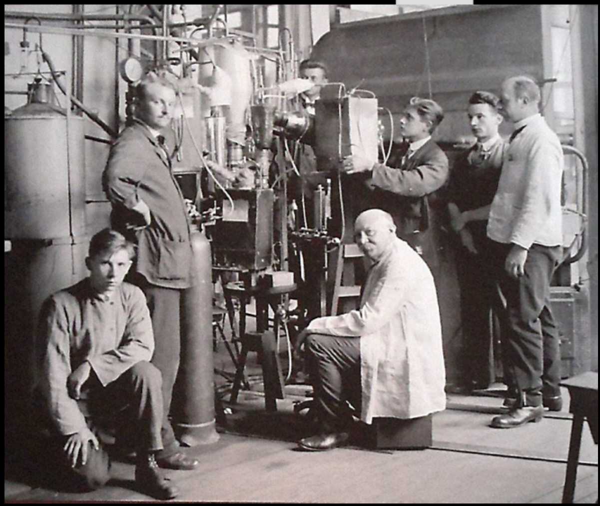
$$\lambda \approx \frac{h}{\sqrt{mk_B T}}$$

- Compare λ against the average distance between atoms, d:

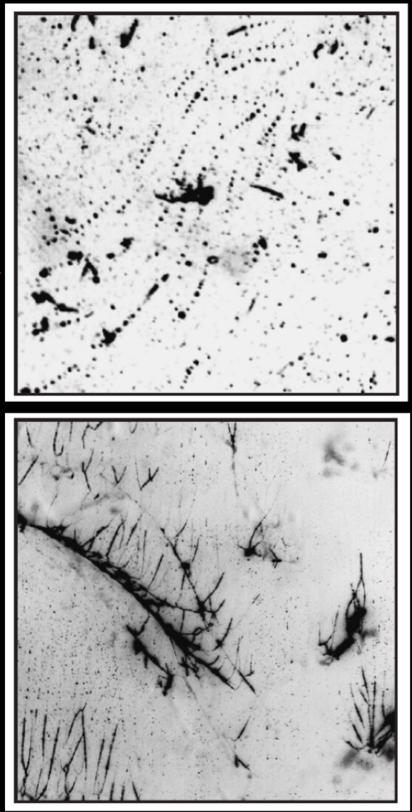


BEC occurs when $\lambda \approx d$

$$\Gamma = \oint_C \mathbf{v} \cdot d\mathbf{l} \in \mathbb{R}$$



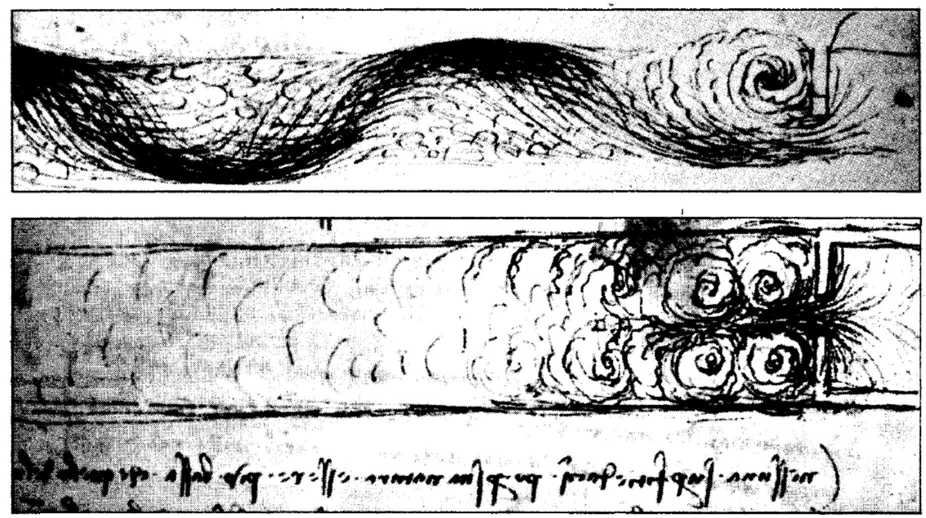
$$\Gamma = \oint_C \mathbf{v} \cdot d\mathbf{l} = \frac{2\pi\hbar}{m} n$$



Paoletti *et al.*, 2008

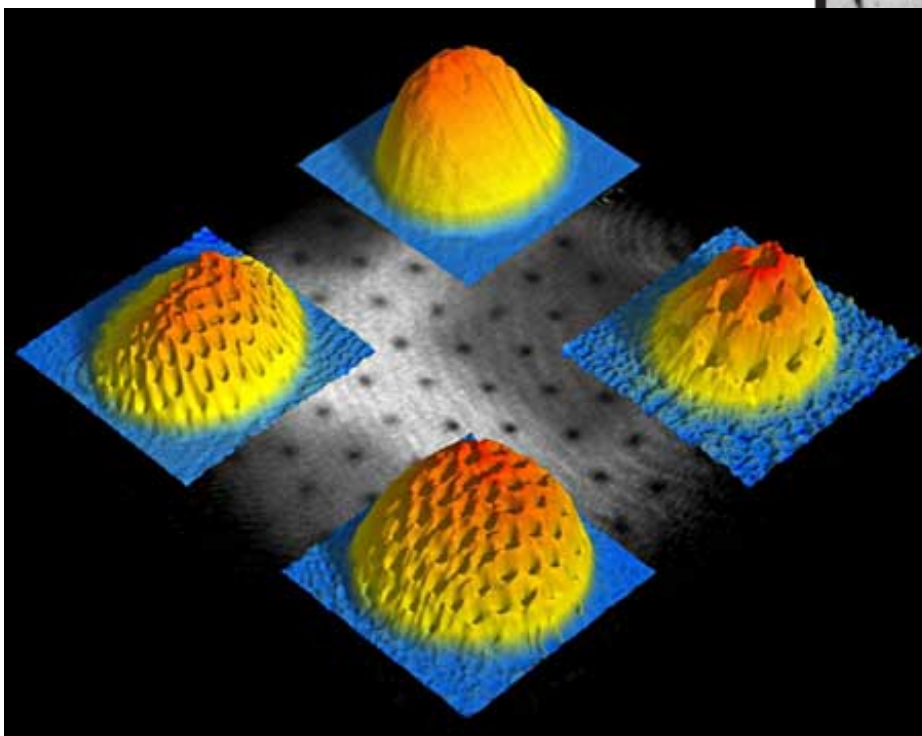
Kuchemann:

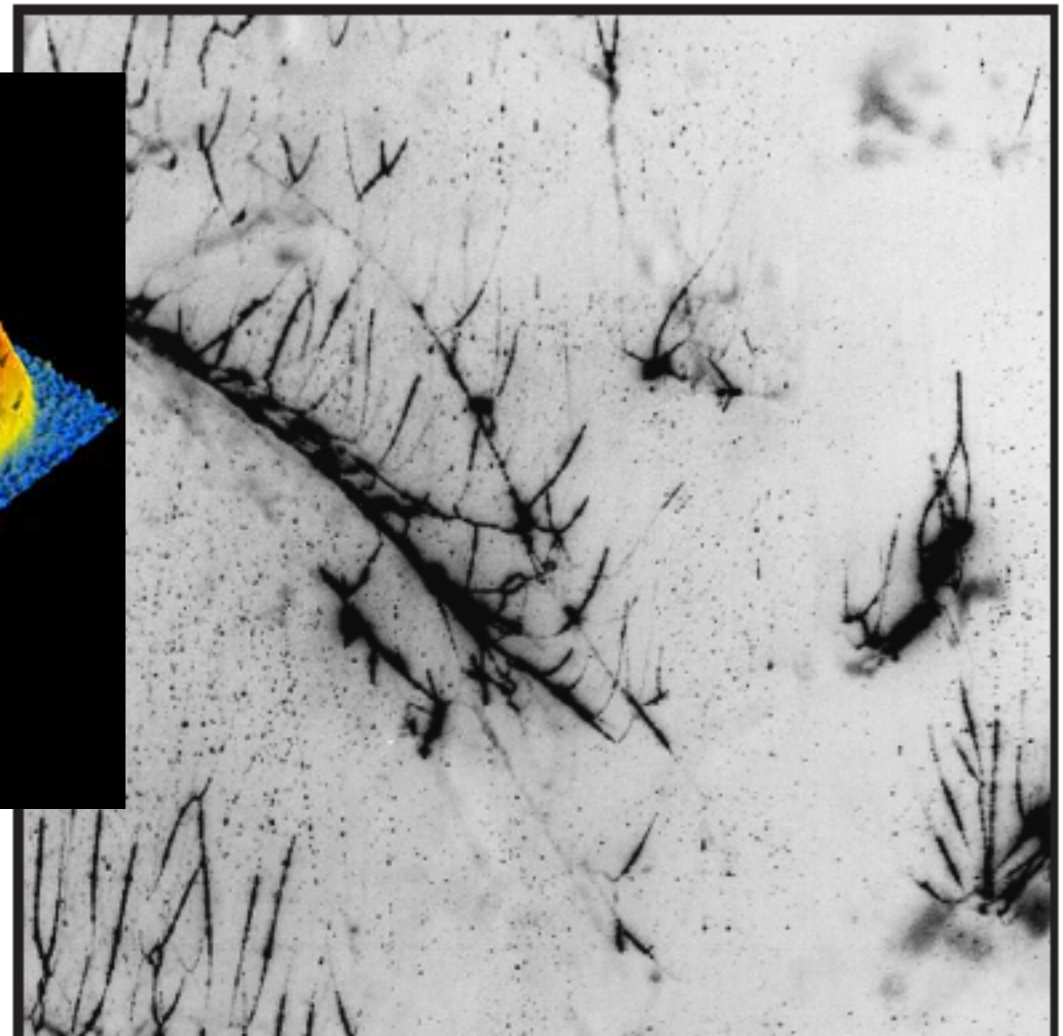
"vortices are the sinews and muscles of fluid motions"



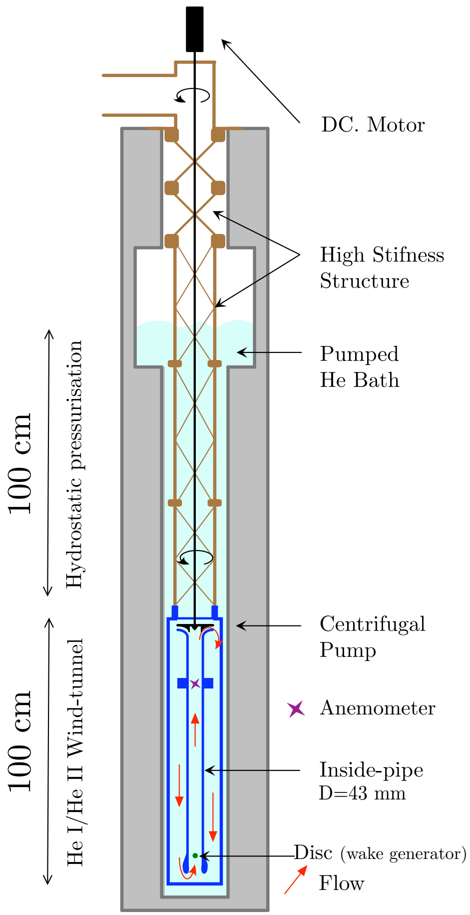
These two drawings from Leonardo's studies in hydrodynamics, now in the collection of the Library of the Institut de France, represent vortex formation with the flow of water around an obstacle or through an opening in a partition within a trough. The second figure of symmetric counter-rotating vortices brings to mind Theodore von Karman's vortex street of asymmetric counter-rotating vortices formed in the wake of a circular cylinder moving through a field. In his 1954 **Aerodynamics,** *von Karmann wrote: "I do not claim to have discovered these vortices: they were known long before I was born. The earliest picture in which I have seen them is one in a church in Bologna, Italy, where St. Christopher is shown carrying the child Jesus across a flowing stream. Behind the saint's naked foot the painter indicated alternating vortices."*

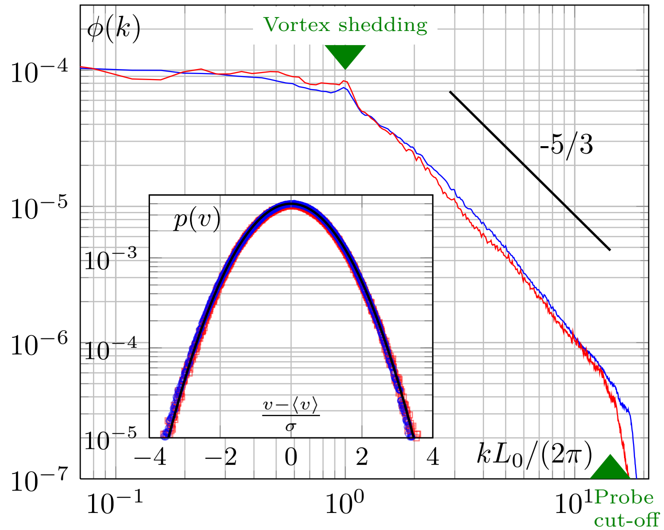
If this is true then Quantum Turbulence represents the 'skeleton'

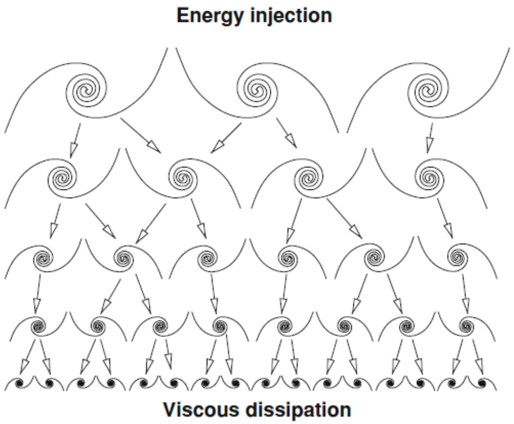




Yet we still see ‘classical’ behaviour

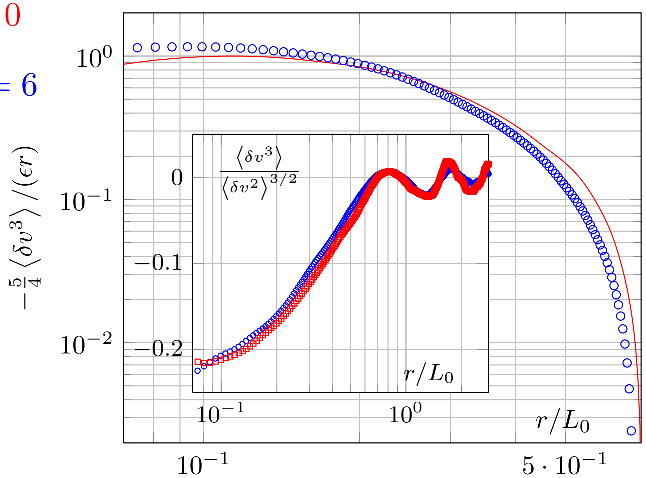






$T = 2.2K, \quad \rho_s/\rho_n = 0$

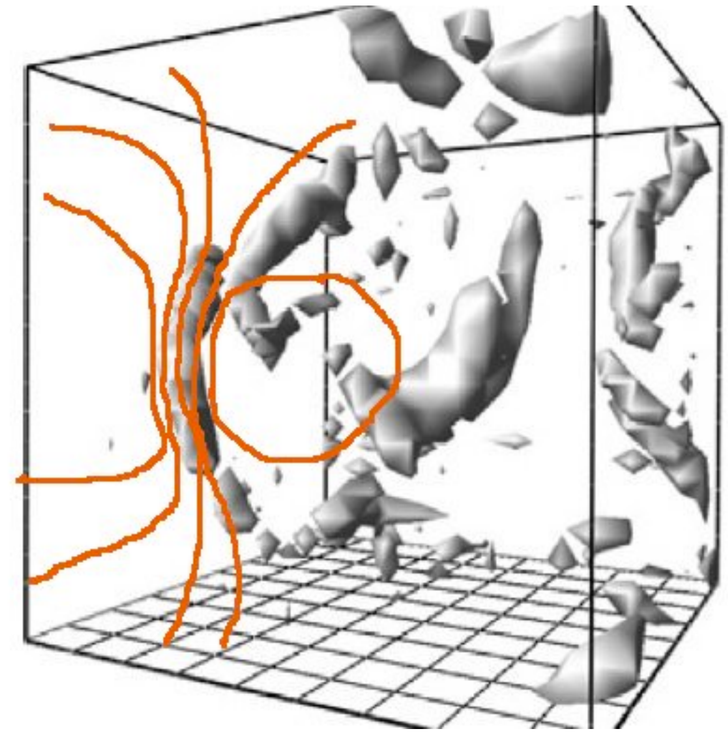
$T = 1.56K, \quad \rho_s/\rho_n = 6$



Salort *et al.*, 2011

Coherent structures

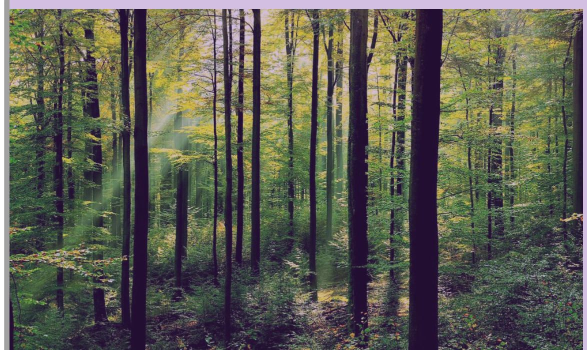
- In classical turbulence vorticity is concentrated in vortical ‘worms’ (She & al, Nature, 1990 ; Goto, JFM, 2008)
- Are there vortex bundles in quantum turbulence ?
- Would allow a mechanism for vortex stretching, i.e. stretch the bundle.



$$\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega} \cdot \nabla)\mathbf{v} + \nu \nabla^2 \boldsymbol{\omega}$$

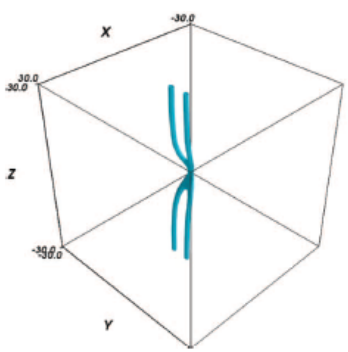
Mathematical approach

3 distinct scales/numerical approaches

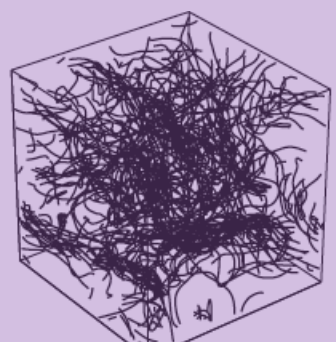


Barenghi *et al.* (2014)

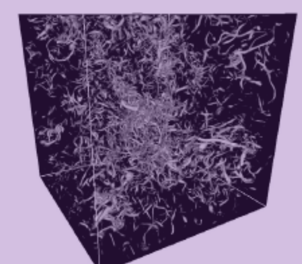
Gross-Pitaevskii



Point Vortex/VFM



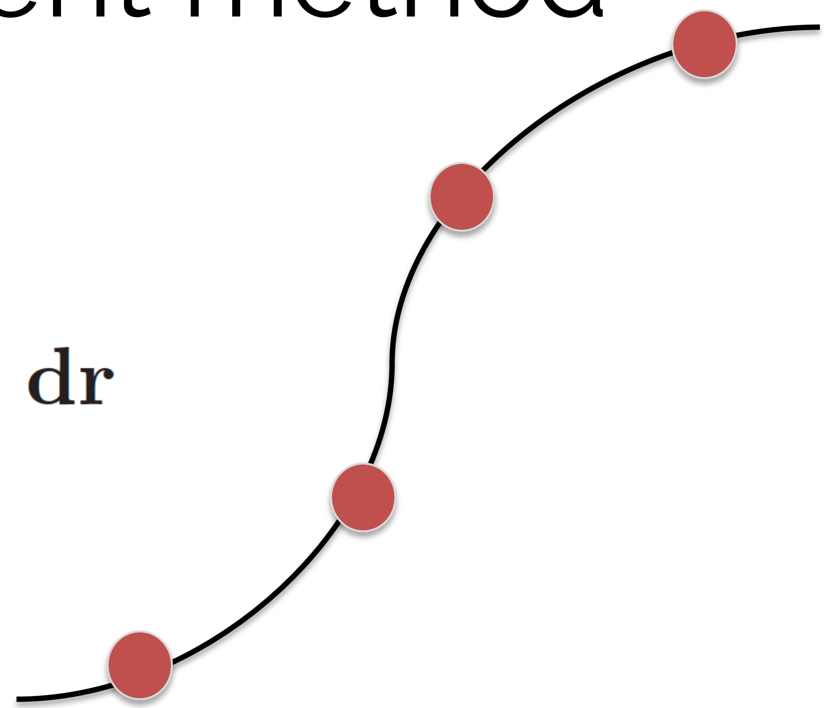
Course-Grained HVBK

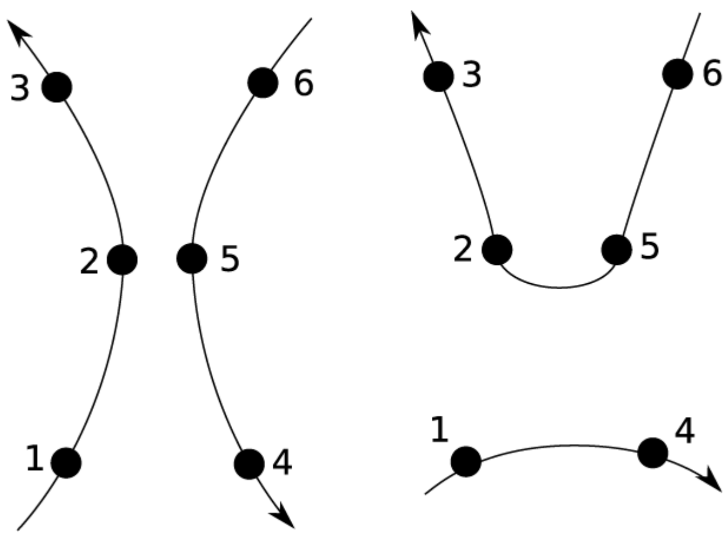


Vortex filament method

Biot-Savart Integral

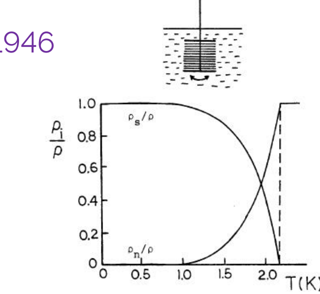
$$\frac{d\mathbf{s}}{dt} = -\frac{\Gamma}{4\pi}\oint_{\mathcal{L}} \frac{(\mathbf{s}-\mathbf{r})}{|\mathbf{s}-\mathbf{r}|^3} \times \mathbf{dr}$$





Model reconnections algorithmically 'cut and paste'

Andronikashvili, 1946



Mutual friction

$$\frac{d\mathbf{s}}{dt} = \mathbf{v}_s^{\mathrm{tot}} + \alpha \mathbf{s}' \times \left(\mathbf{v}_n^{\mathrm{ext}} - \mathbf{v}_s^{\mathrm{tot}}\right) - \alpha' \mathbf{s}' \times \left[\mathbf{s}' \times \left(\mathbf{v}_n^{\mathrm{ext}} - \mathbf{v}_s^{\mathrm{tot}}\right)\right]$$

Counterflow Turbulence

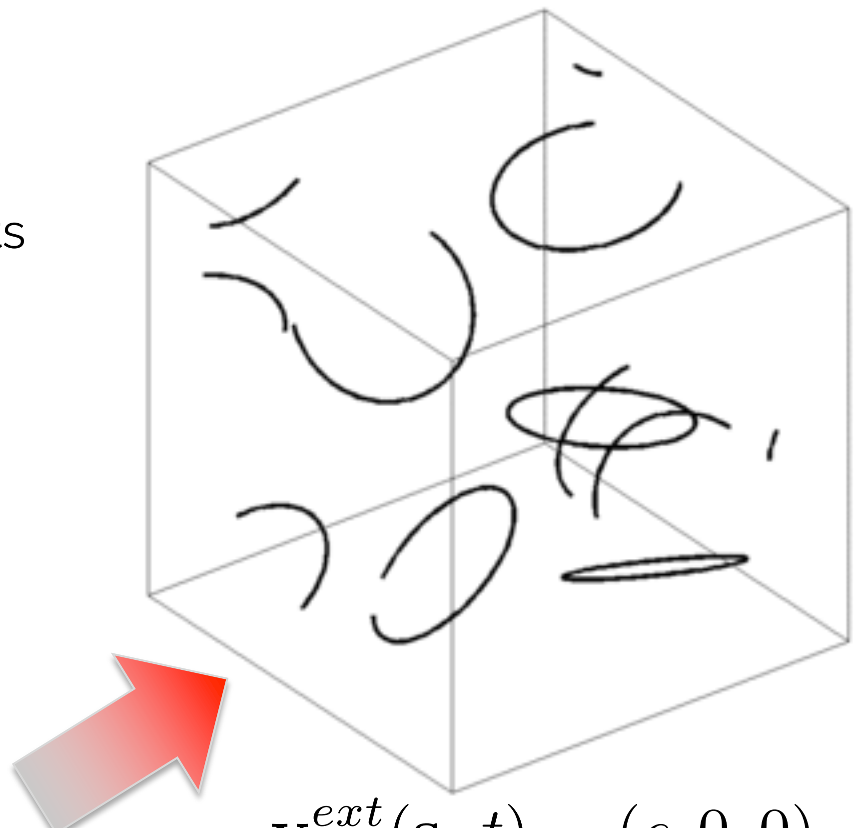
Normal viscous fluid coupled to inviscid superfluid via mutual friction.

Superfluid component extracts energy from normal fluid component via Donelly-Glaberson instability, amplification of Kelvin waves.

Kelvin wave grows with amplitude:

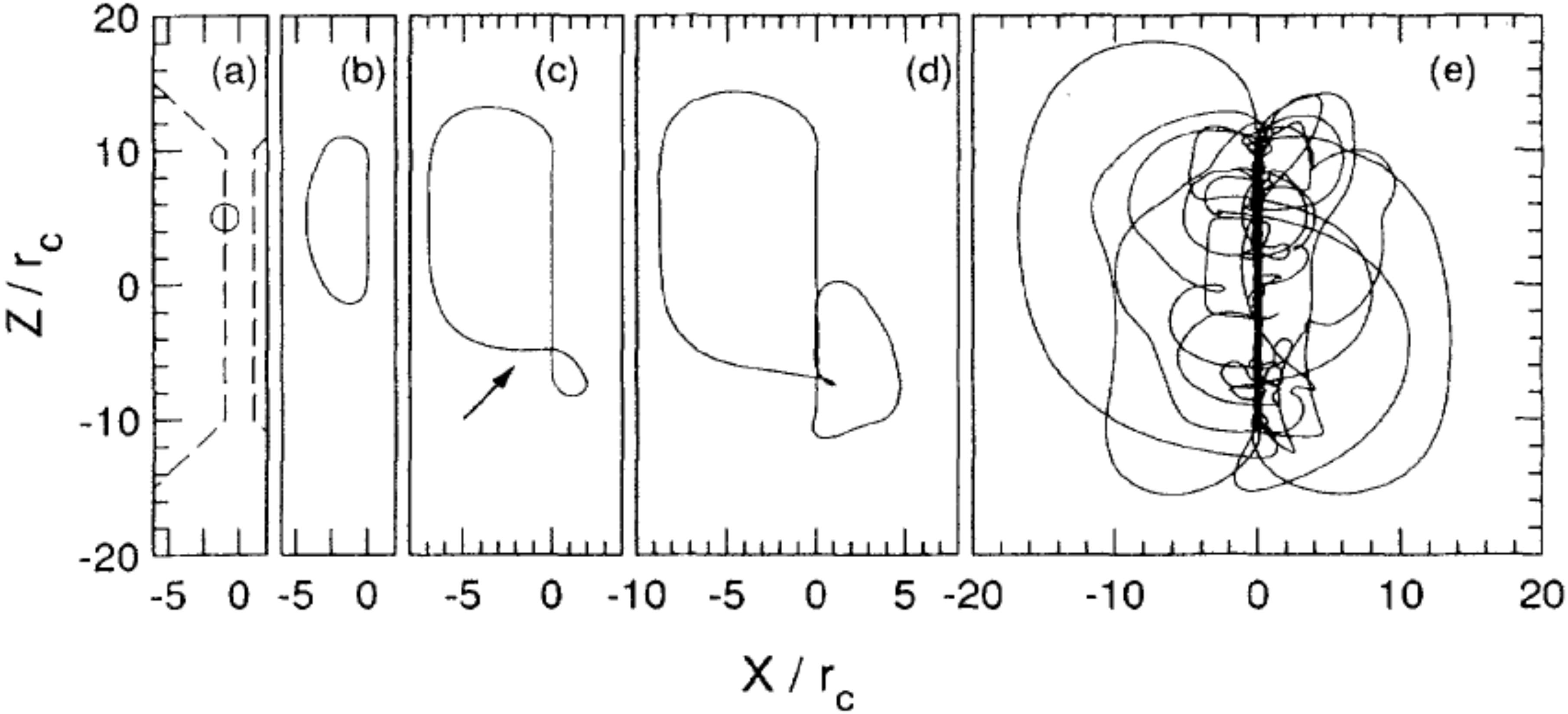
$$A(t) = A(0)e^{\sigma t}$$

$$\sigma(k) = \alpha(kV - \nu' k^2)$$



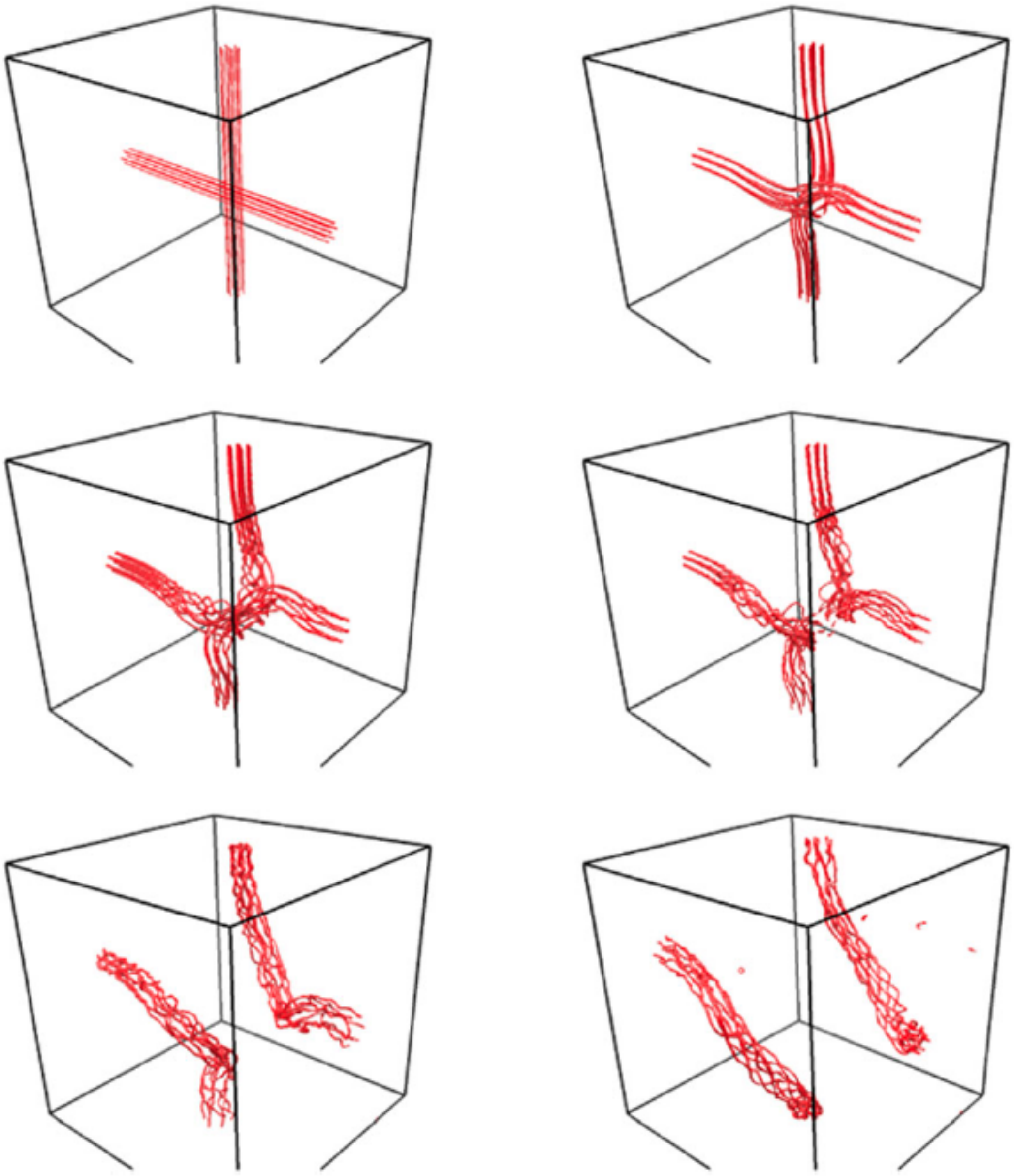
$$\mathbf{v}_n^{ext}(\mathbf{s},\, t) = (c, 0, 0)$$

Generation of bundles at finite temperatures



[illegible]

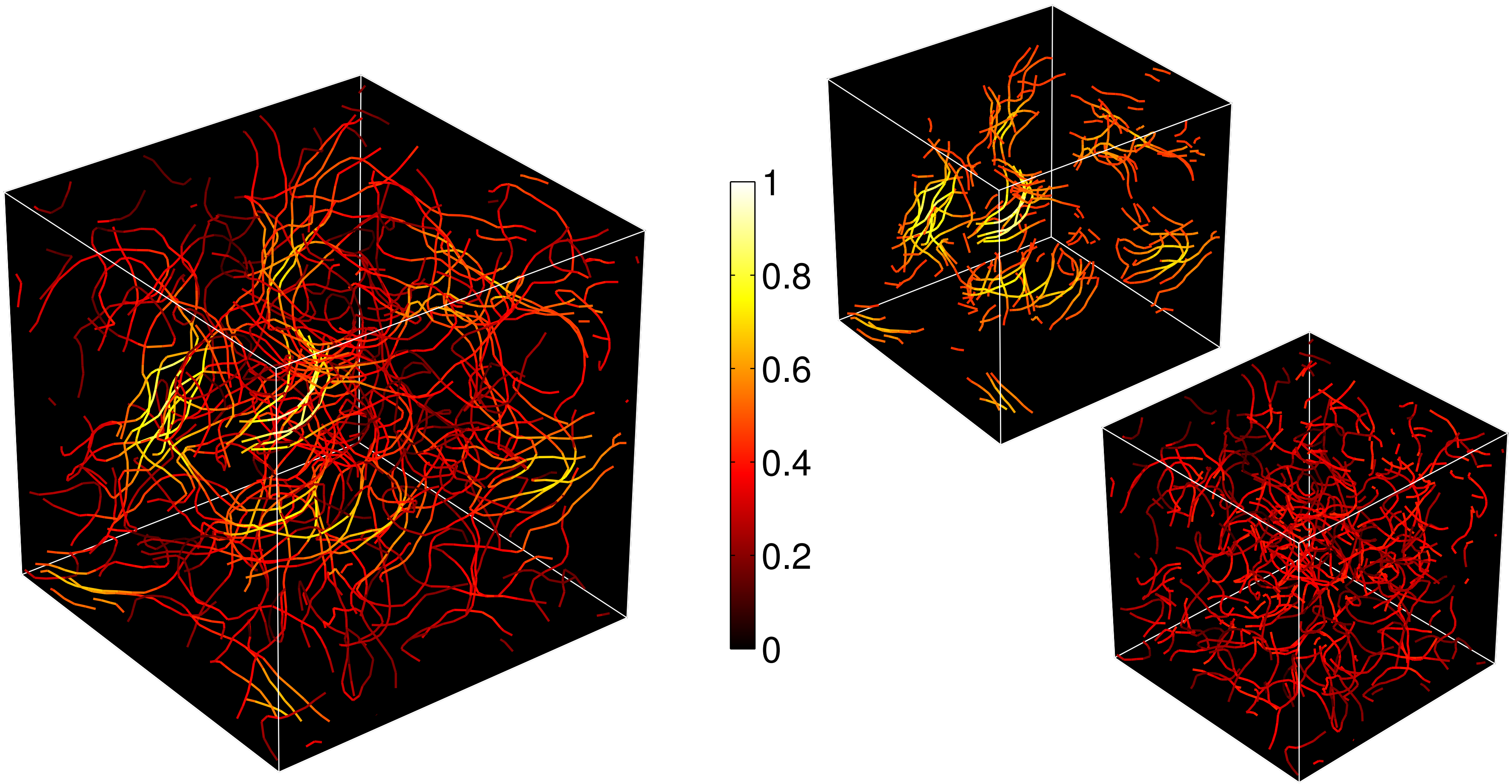
Reconnections: Bundles remain intact



Numerical simulations using both GPE and vortex filament method.

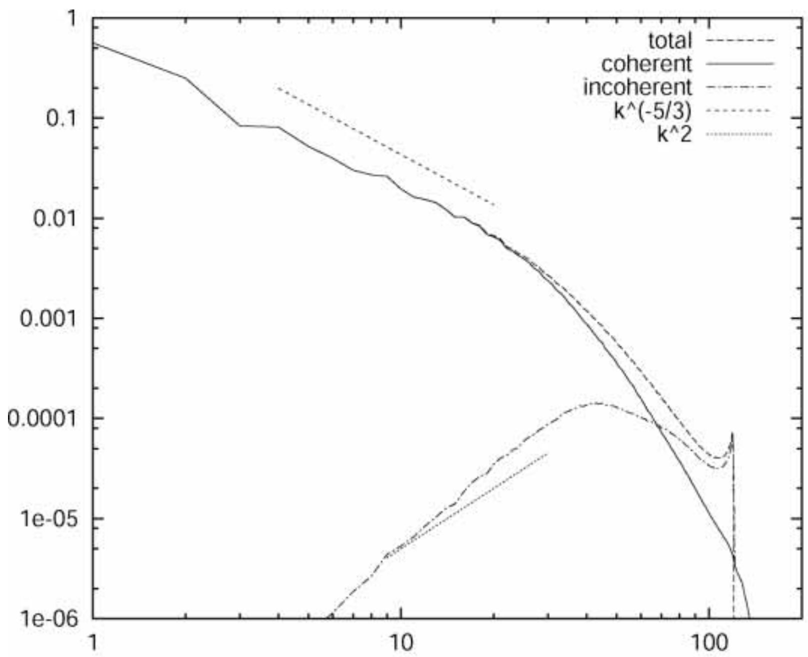
Alamri *et al.* 2008

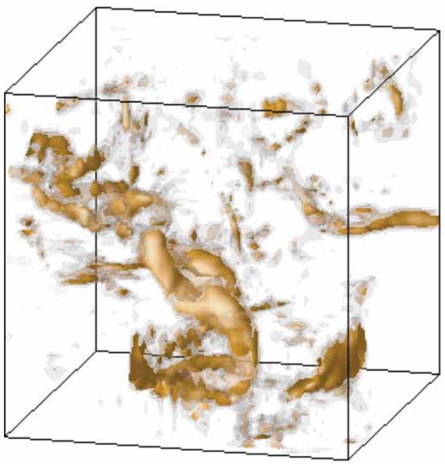
Decomposition of a tangle

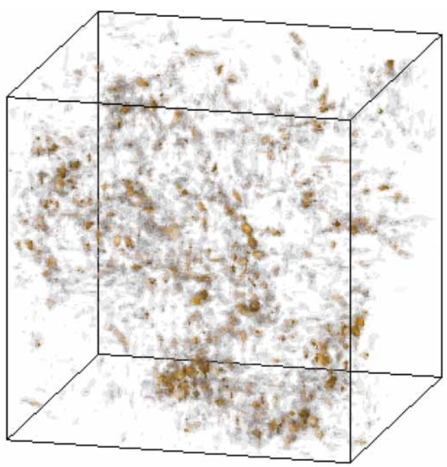


AWB, Laurie & Barenghi, 2012

Motivation

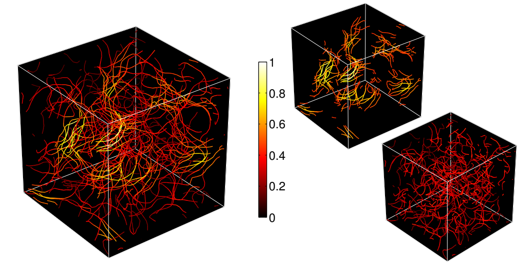


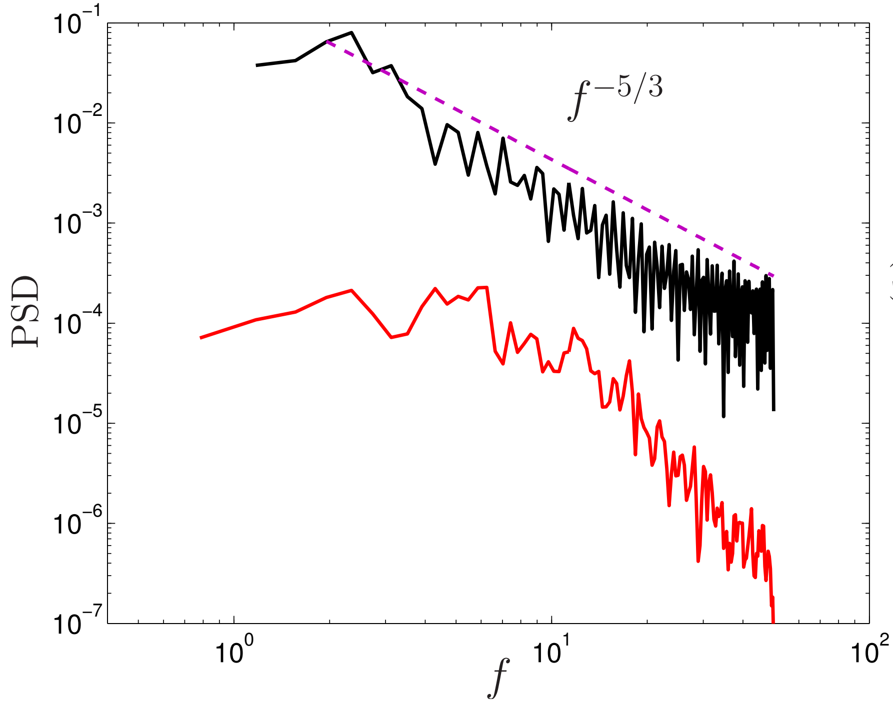


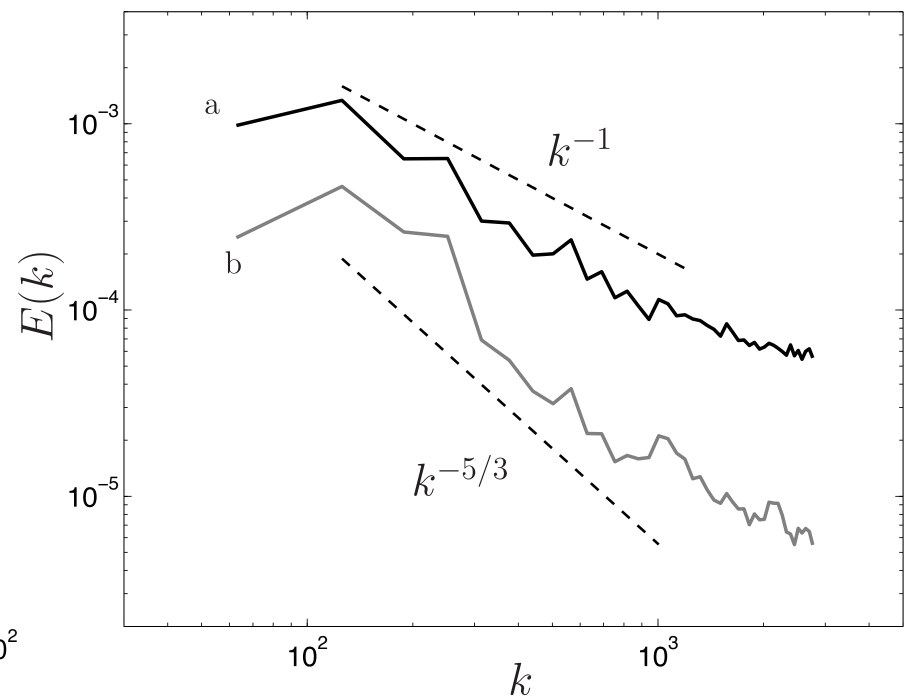


Roussel, Schneider & Farge, 2005

Numerical results



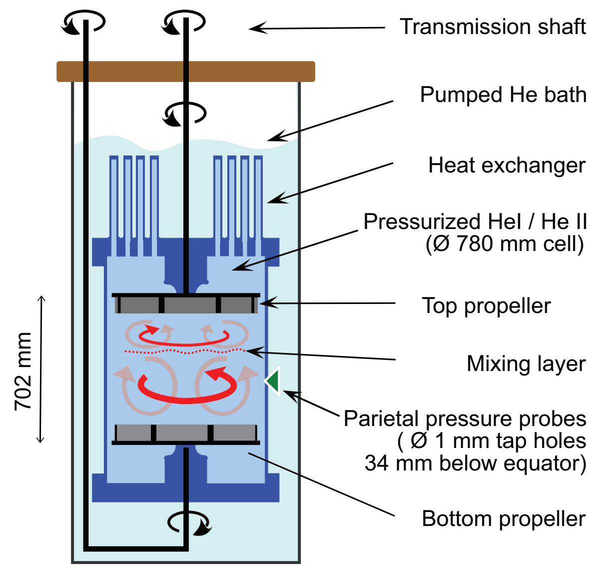


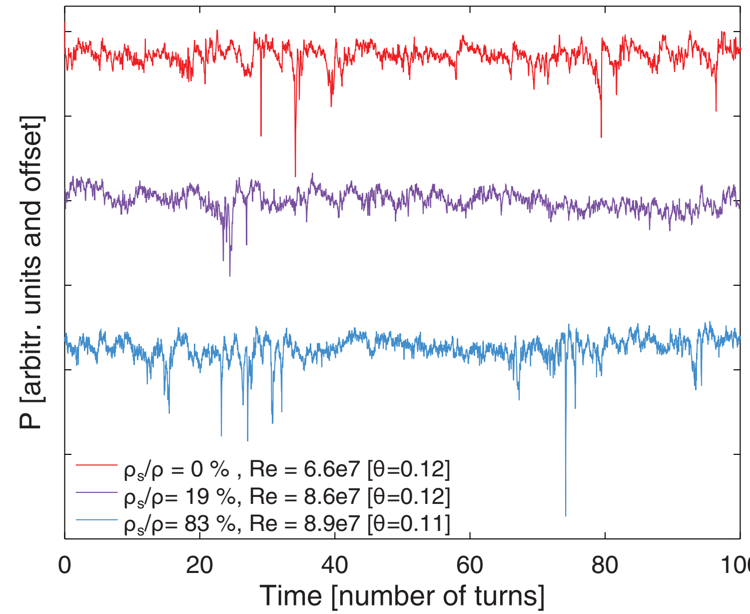


Left, frequency spectra (red polarised ; black total), right energy spectrum, upper random component, lower polarised component.

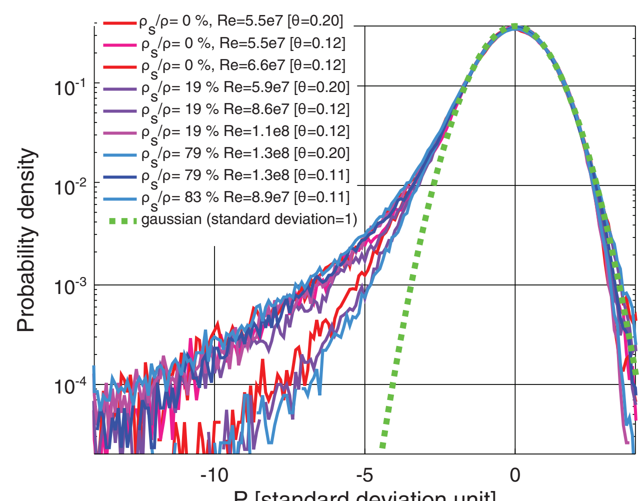
Experimental detection

Rusaouen *et al.*, 2017





Presence of coherent structures inferred from intermittent pressure drops



Hall-Vinen-Bekarevich-Khalatnikov Equations

Course-grained, macroscopic model

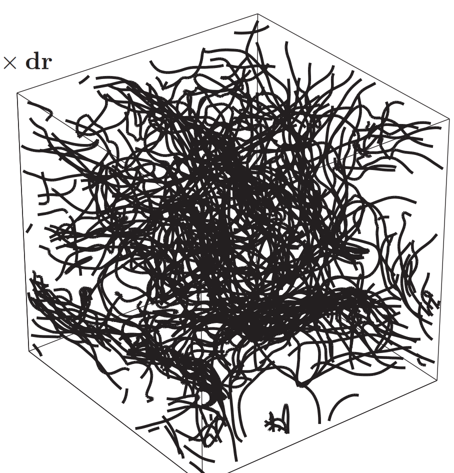
$$\frac{\partial \mathbf{v}_n}{\partial t} + (\mathbf{v}_n \cdot \nabla)\,\mathbf{v}_n = -\frac{1}{\rho}\nabla P + \mu \nabla^2 \mathbf{v}_n + \frac{\rho_s}{\rho}\mathbf{F}, \quad \nabla \cdot \mathbf{v}_n = 0,$$

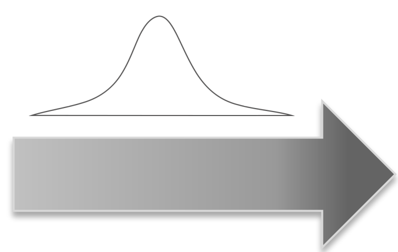
$$\frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \cdot \nabla)\,\mathbf{v}_s = -\frac{1}{\rho}\nabla P - \frac{\rho_n}{\rho}\mathbf{F}, \quad \nabla \cdot \mathbf{v}_s = 0.$$

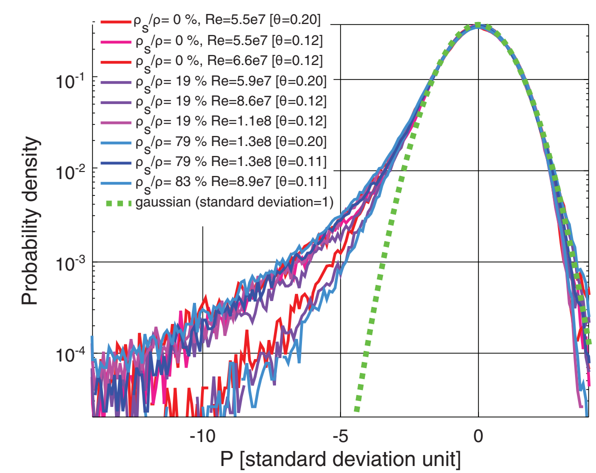
$$\mathbf{F} \simeq \alpha \rho_s \langle |\omega_{\mathbf{s}}| \rangle (\mathbf{v}_s - \mathbf{v}_n)$$

$$\rho_s \gg \rho_n : \qquad \nabla^2 P \sim \frac{\rho_s}{2}(\omega_s^2 - \sigma_s^2)$$

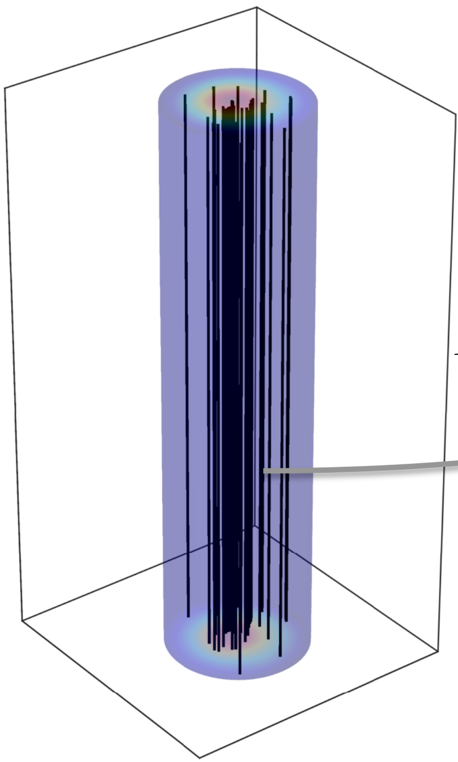
$$\frac{d\mathbf{s}}{dt} = -\frac{\Gamma}{4\pi}\oint_{\mathcal{L}} \frac{(\mathbf{s}-\mathbf{r})}{|\mathbf{s}-\mathbf{r}|^3} \times d\mathbf{r}$$







A single bundle in isolation



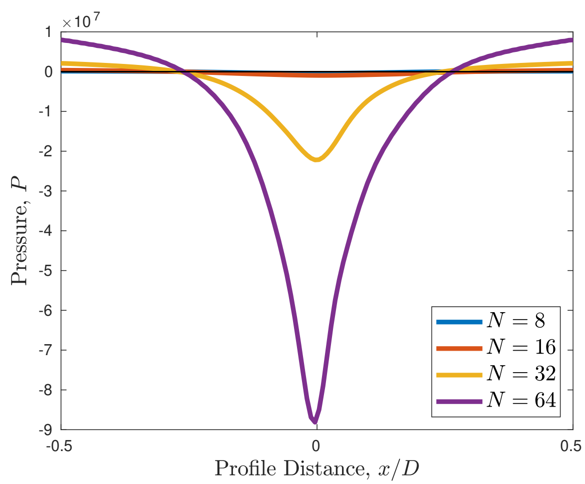
$$\hat{F}(|\mathbf{k}|) = \exp\left(-\frac{|\mathbf{k}|^2}{2k_f^2}\right)$$

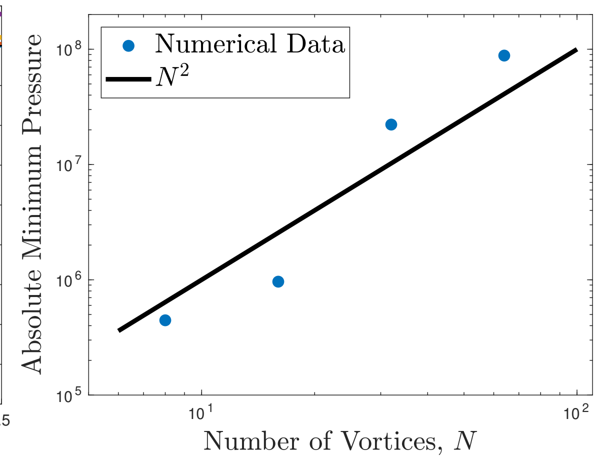
$$\cancel{\frac{\partial \mathbf{v}_s}{\partial t}} + (\mathbf{v}_s \cdot \nabla)\,\mathbf{v}_s = -\frac{1}{\rho}\nabla P - \cancel{\frac{\rho_n}{\rho}}\mathbf{F}$$

$$\mathbf{v}_s = (v_r, v_\theta, v_z) = \left(0, \frac{N\Gamma}{2\pi r}, 0\right)$$

$$P = P_0 - \frac{\rho_s N^2 \Gamma^2}{8\pi^2 r^2},$$

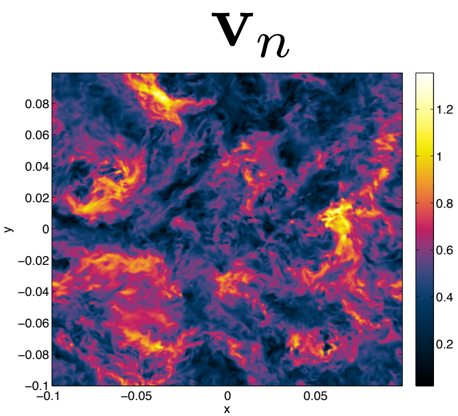
$$\min_V P(N) \sim -N^2$$



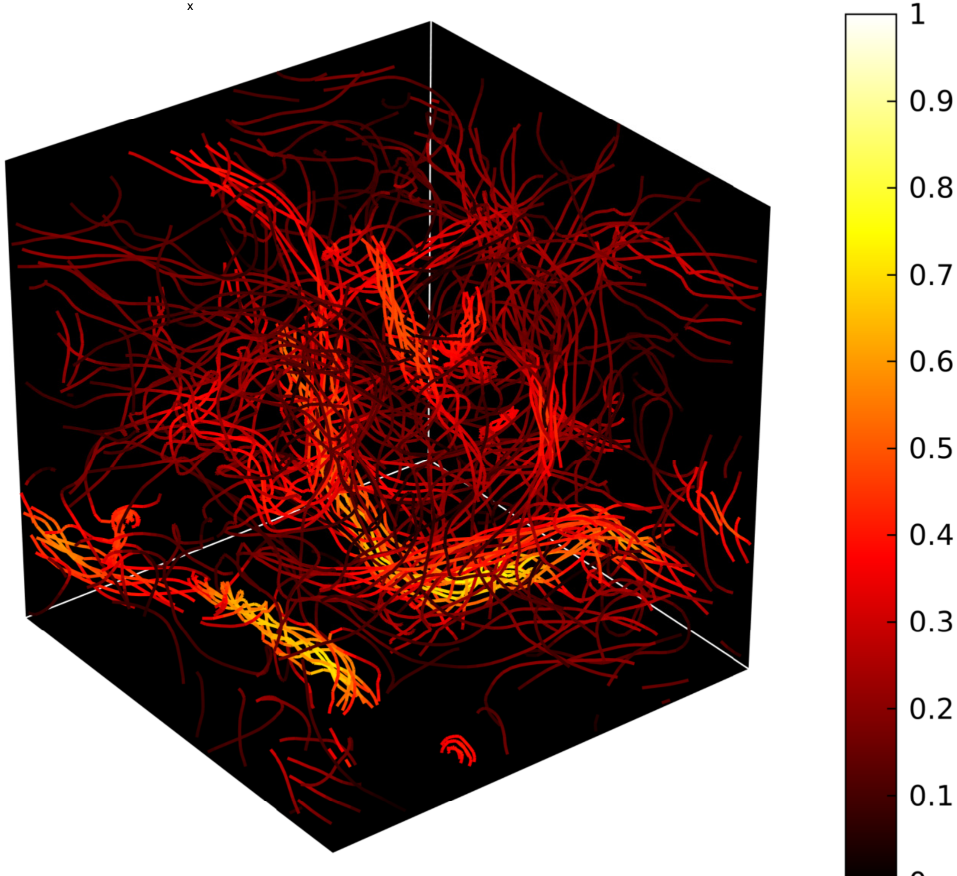


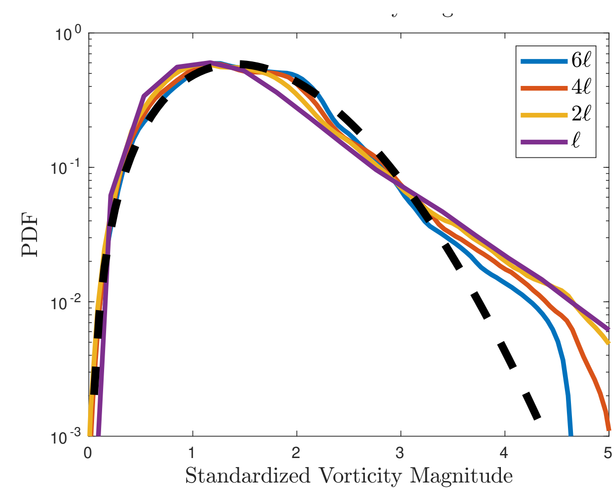
AWB & Laurie arxiv:1910.00276

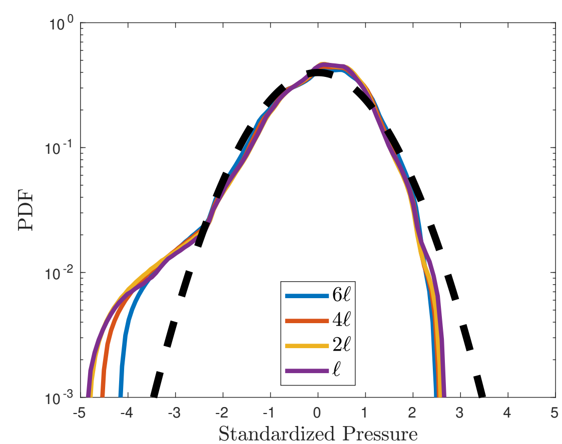
Turbulent Tangle

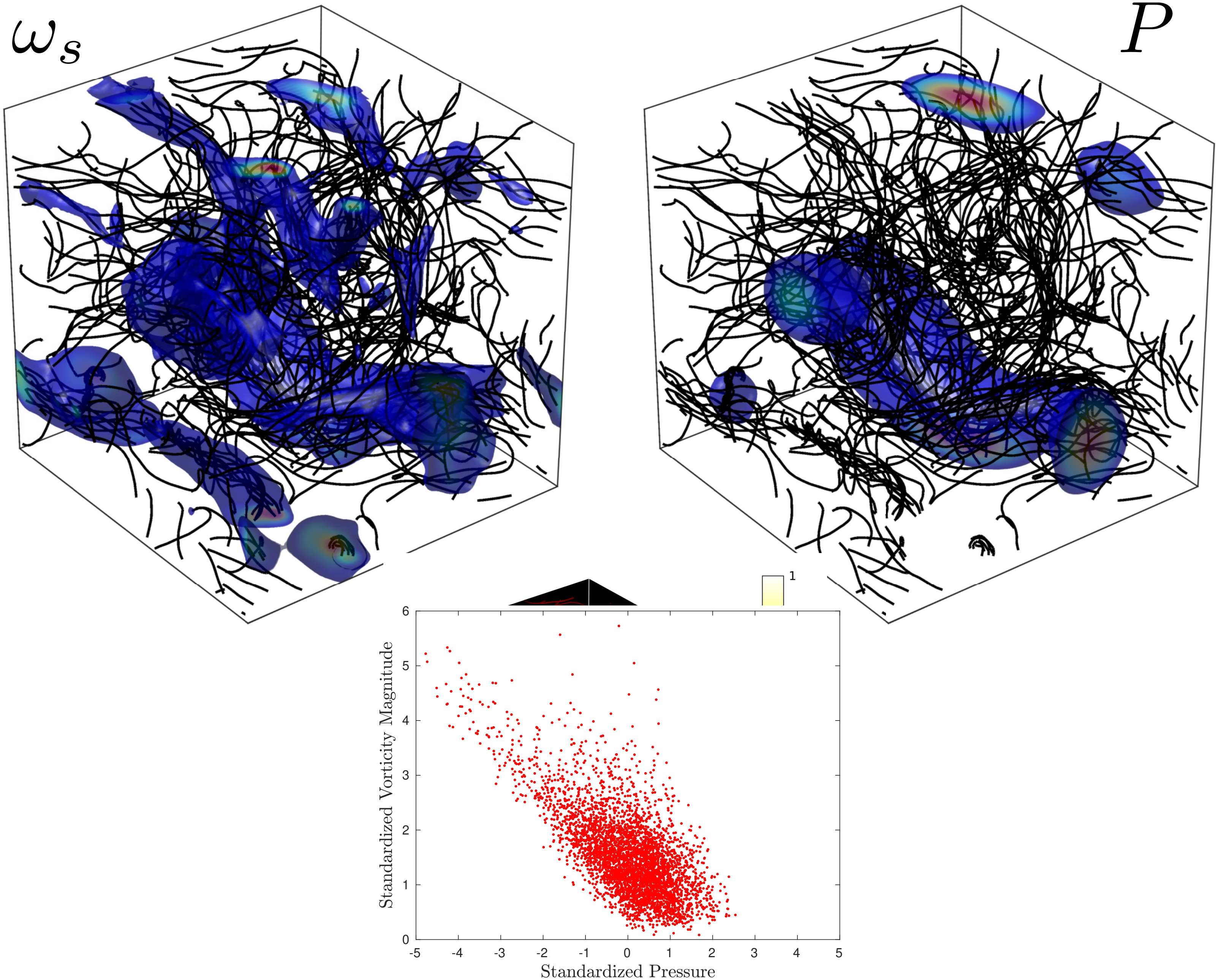


$$\hat{F}(|\mathbf{k}|) = \exp\left(-\frac{|\mathbf{k}|^2}{2k_f^2}\right) \qquad k_f = 2\pi/l_f$$



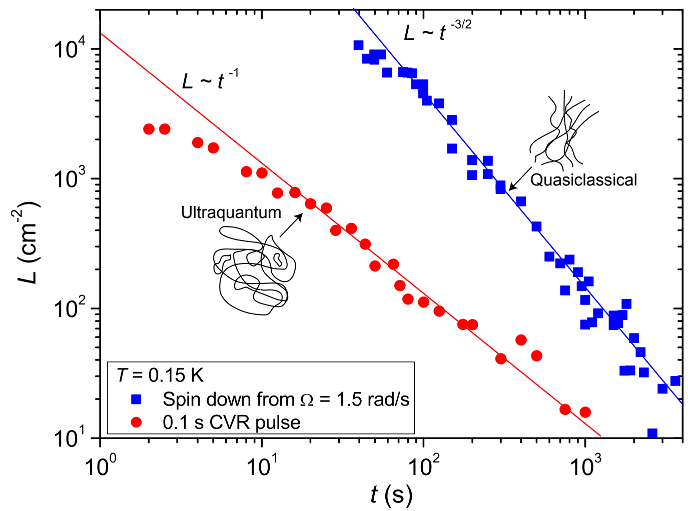


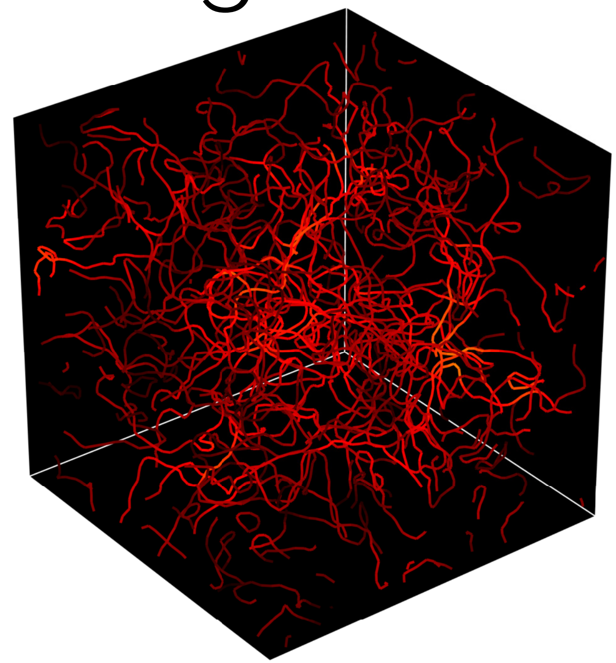




ω_s
P
1
Standardized Vorticity Magnitude
Standardized Pressure
0
1
2
3
4
5
6
-5
-4
-3
-2
-1
0
1
2
3
4
5

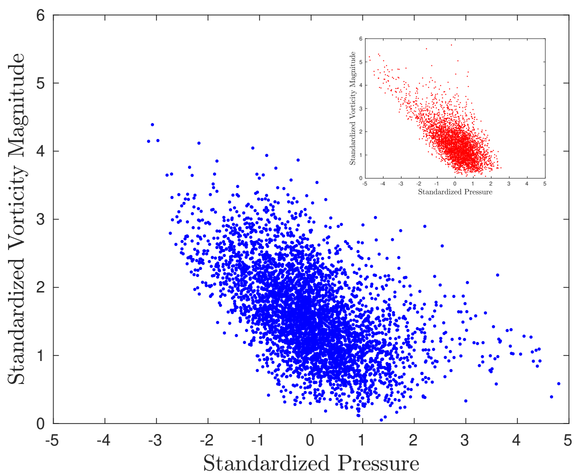
Random 'Vinen' Tangle

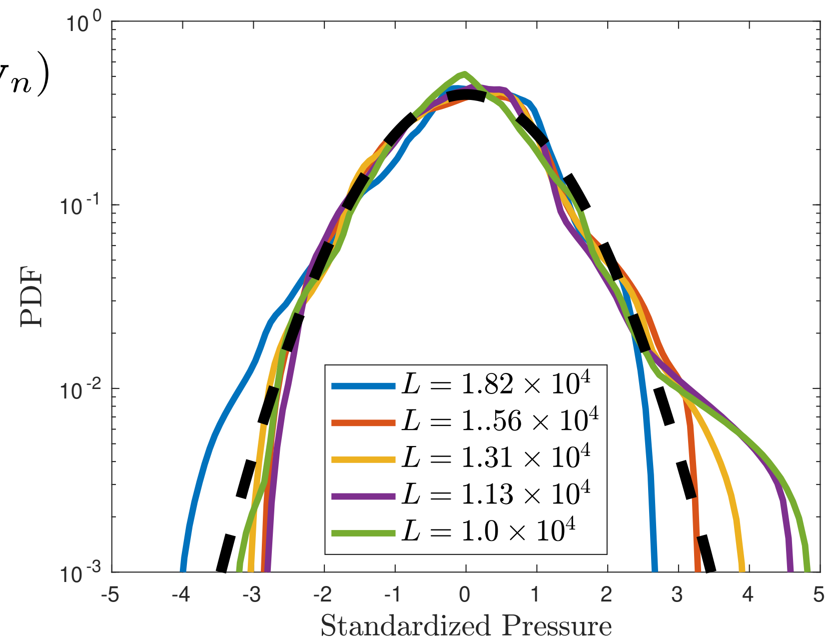




Walmsley *et al.* 2013

$$\mathbf{F} \simeq \alpha \rho_s \langle |\omega_{\mathbf{s}}| \rangle (\mathbf{v}_s - \mathbf{v}_n)$$





Summary

- Coherent vortical structures are present in the quasi-classical regime of Quantum Turbulence.

- Important (essential?) for K41 like statistical properties of QT.

- Good agreement between macroscopic HVBK model and mesoscale vortex approach.

- Interesting high pressure signal found in the Vinen regime.

The End