# Extreme events in lasers

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Wave Turbulence in Nonlinear Optics, BECs, and Related Areas

# Collaborators

1. Theory
   - M. Brambilla, L. Columbo (Università e Politecnico di Bari, Italy)
   - F. Prati (Università dell'Insubria, Como, Italy)
   - C. Rimoldi (Université Côte d'Azur, INPHYNI, Nice, France)
2. Experiment
   - S. Barland, f. Gustave, P. Walczak (Université Côte d'Azur, CNRS, INPHYNI, Nice, France)

# Outline

# Extreme events

Extreme events in nature and society (natural disasters, market crashes, pandemics etc.)

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statistical and dynamical approach

- possible analogies in different contexts
- generating mechanisms and predictability

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**What are extreme events?**

Events that lie in the tail of a probability distribution, presenting a deviation from the global data behavior (ex. heavy tails). [Jentsch 2006, Springer]

# A particular kind of extreme events

## Rogue waves

Isolated high-peak **oceanographic** extreme events that appear and disappear into nothingness.
$\rightarrow$ More frequent than expected by gaussian statistics.
[Kharif 2009, Springer]

$\downarrow$

Formal analogy with **fiber optics** through the Nonlinear Schrödinger Equation.
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two main branches of study for extreme events:

- passive dispersive conservative (or weakly dissipative) systems (fibers).
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with spatial degrees of freedom.

# Extreme events in active dissipative optical systems

Lack of a formal analogy with other extreme event contexts.

$\downarrow$

Exploration of extreme events and possible analogies
from a statistical perspective.

## Focus

- dynamical generating mechanisms (ex. spatiotemporal chaos [Selmi 2016, PRL], vortex turbulence [Gibson 2016, PRL], external crisis [Zamora-Munt 2013, PRA]).
- Extreme event predictability [Zamora-Munt 2013, PRA], [Alvarez 2017, EPJST].

# Localized structure interaction

Conservative case: localized structure interaction

↓

possible mechanism for the formation of extreme events.

[Frisquet 2013, PRX]

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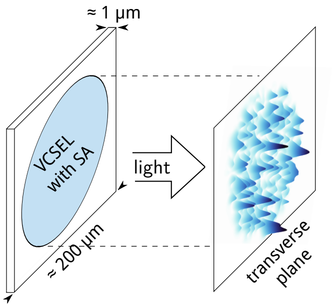
## Dissipative case

Motivation to study, in this context:

- cavity soliton and phase soliton interaction
- relationship with extreme events

# Broad-area semiconductor LSA

## Setup




- spatially 2D system
- active and passive NL materials
- incoherent pump

- diffraction
- nonlinearity
- saturable absorption
- cavity feedback

$\Rightarrow$ self-pulsing

$\Rightarrow$ modulational instability

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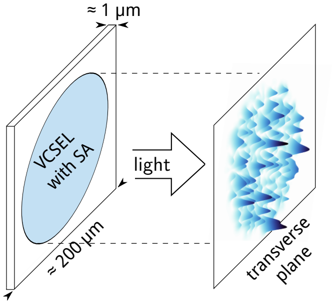


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## Model

[Vahed 2014, Phil. Trans. R. Soc.]

$F \propto$ amplitude electric field $\Leftarrow$ $\dot{F} = [(1 - i\alpha)D + (1 - i\beta)\bar{D} - 1 + (d + i)\nabla_{\perp}^{2}]F$

$D \propto$ amplifier carrier density $\Leftarrow$ $\dot{D} = b[\mu - D(1 + |F|^{2}) - BD^{2}]$

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$$r = \frac{\tau_{amp}}{\tau_{abs}}$$

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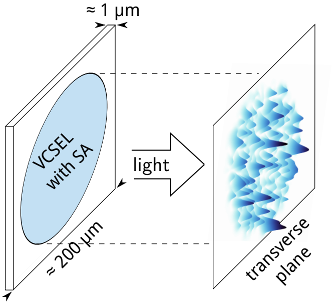


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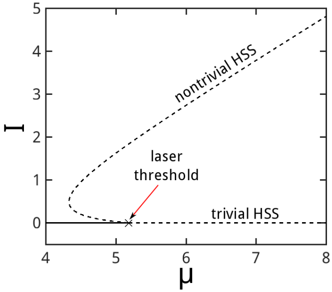


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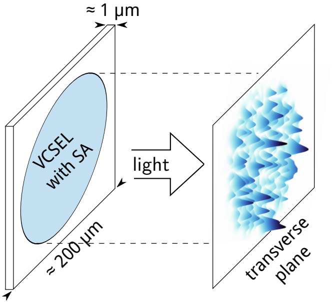


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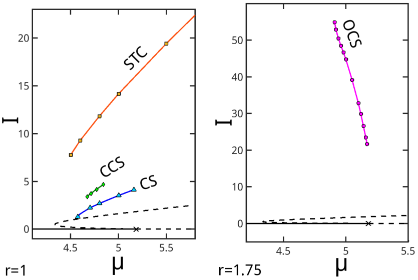


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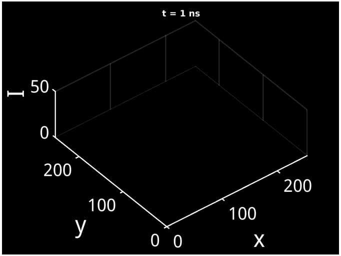
## Stationary cavity soliton

## Oscillatory cavity soliton

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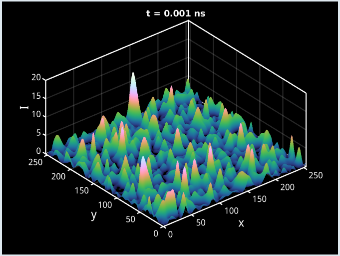


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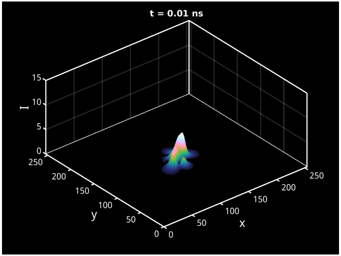

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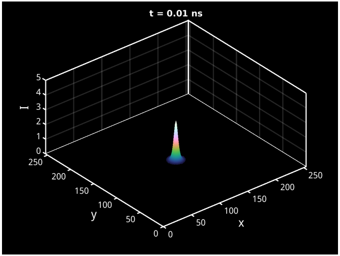


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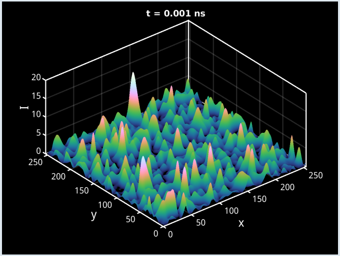


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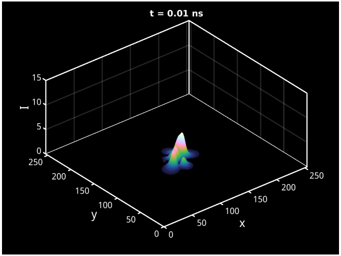

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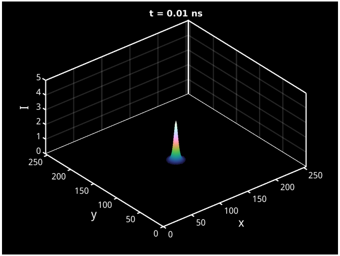


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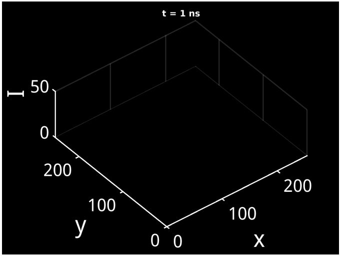


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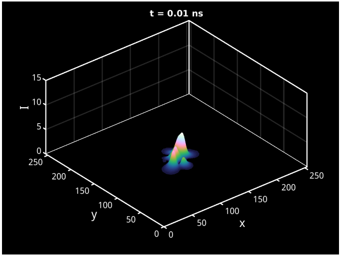

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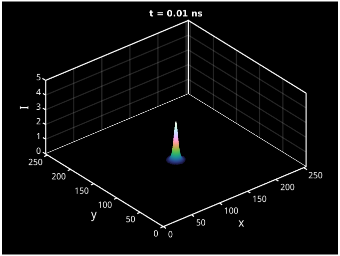


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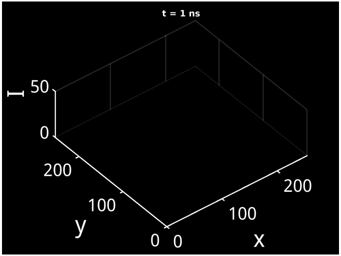


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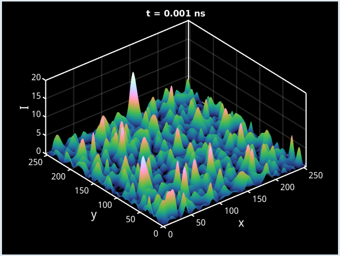
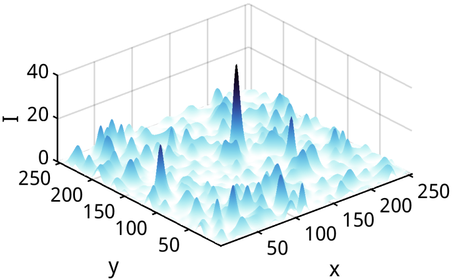


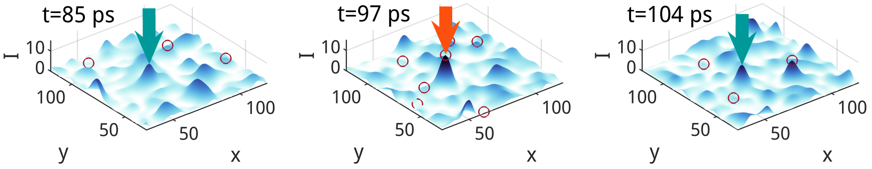

## Chaotic cavity soliton

Search for **extreme events**
in the **spatiotemporal chaos** regime




single **events** (extreme or not)
are represented by the **spatiotemporal maxima**
of the intensity in the transverse plane

## Spatiotemporal maxima selection



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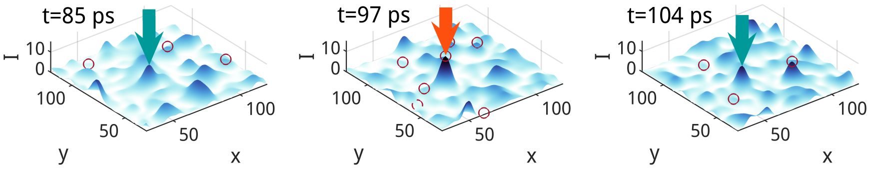


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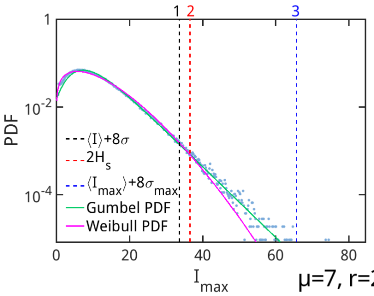


μ=7, r=2.5

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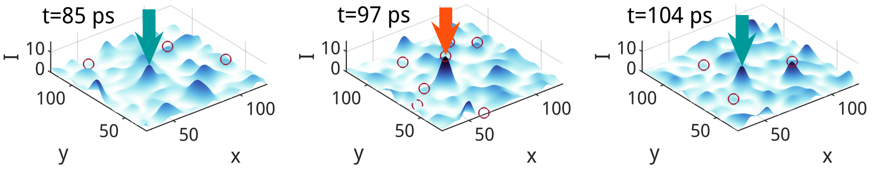


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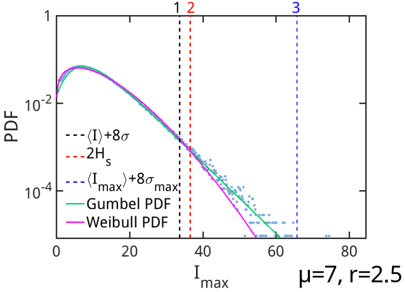
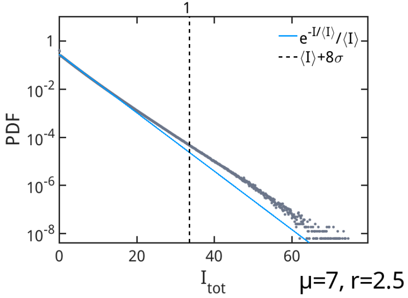



## Total intensity PDF



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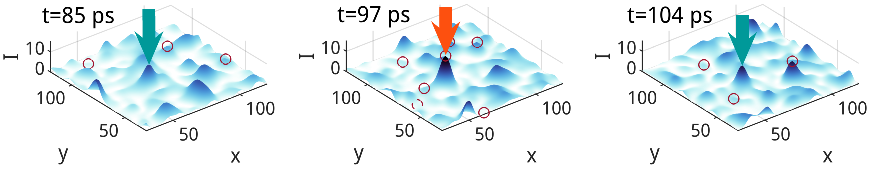


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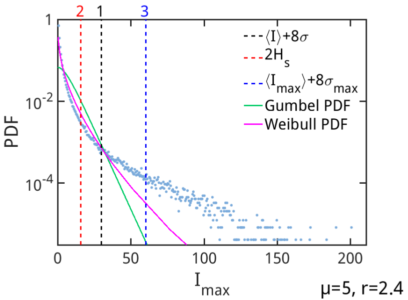


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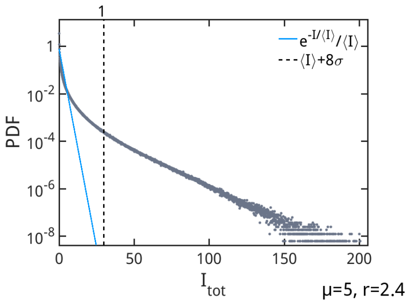
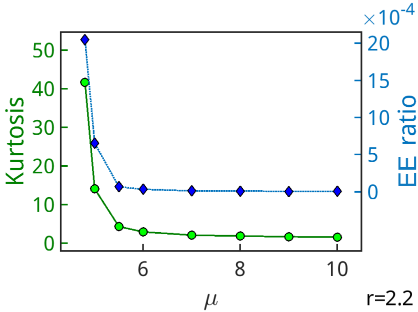


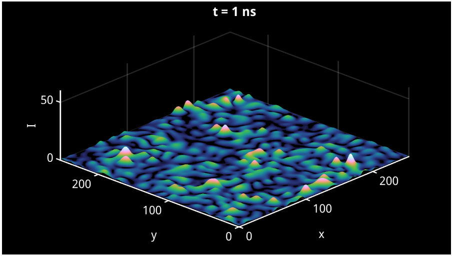
# Extreme event optimization

## EE ratio density plot




- Kurtosis: $\frac{\mu_4}{\sigma^4}$
- EE ratio: $\frac{\#\ \text{EEs}}{\#\ \text{maxima}}$

## Favorable regime for EEs



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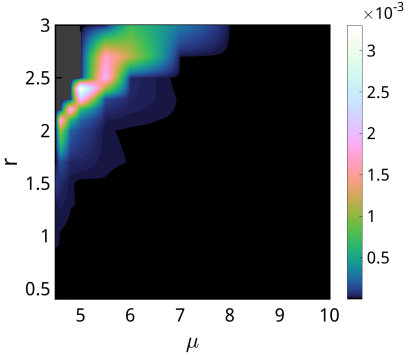


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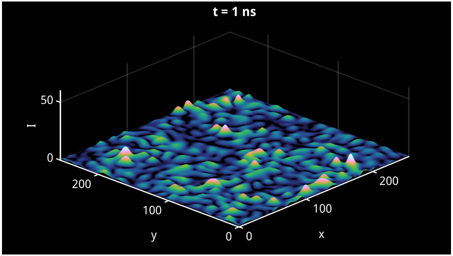

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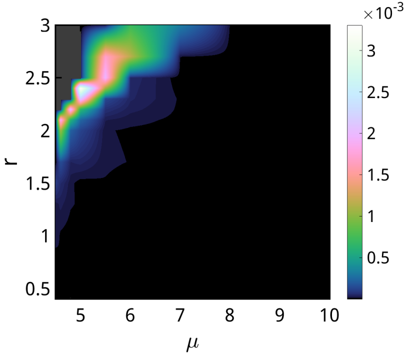


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## Recipe for extreme events

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- fast saturable absorber ($r \geq 2$)

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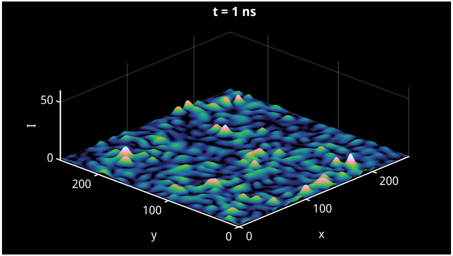

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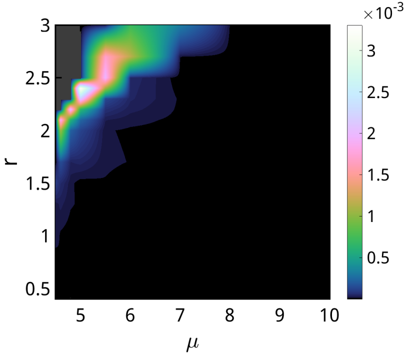


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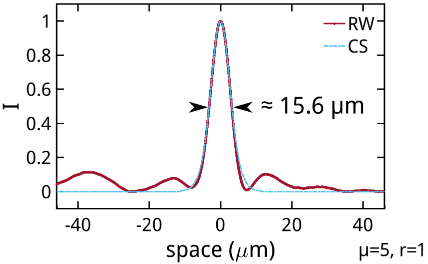
# Comparison with cavity solitons

## Stationary CS

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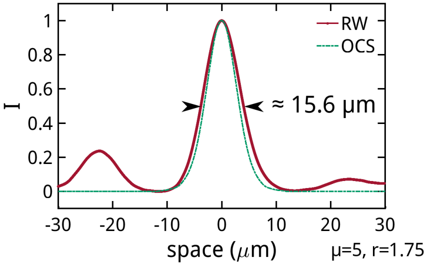

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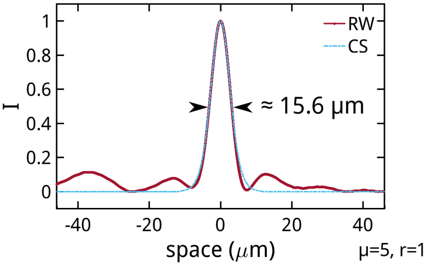


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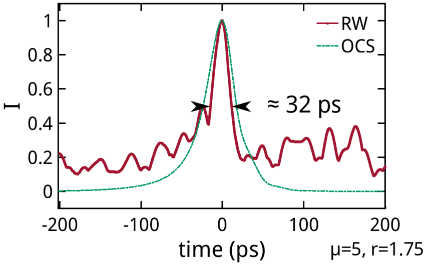

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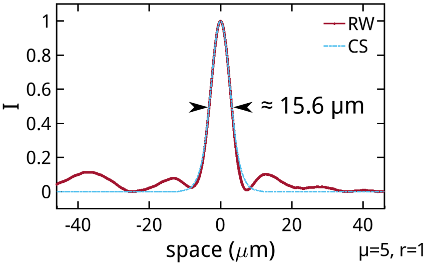


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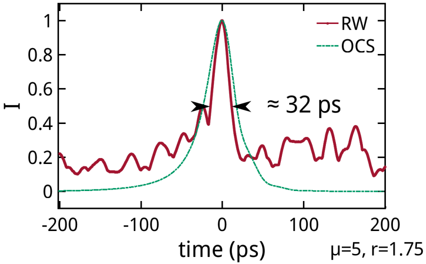



## Role of CSs in extreme event formation

For values of *r* where CSs exist:

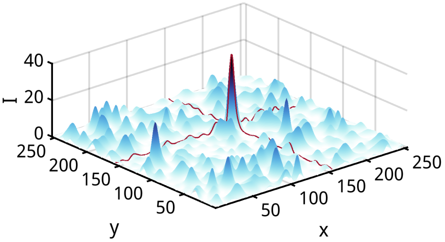
higher EE ratio where there is multistability with the soliton branch

- possible enhancement of EE
- possibly similar generating mechanism (modulational instability in space and Hopf instability on the stationary solutions in time → Q-switching)

# Conclusions

## Results

- ✓ Extreme events in the system (2D+time)
- ✓ EE optimization for low pump and fast SA
- ✓ Relation to cavity solitons




## Open questions

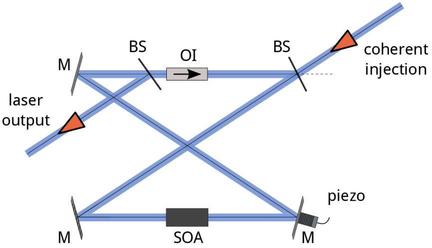
- Predictability
- Conservative limit

C. Rimoldi, F. Prati, S. Barland, G. Tissoni, *Phys. Rev. A* **95**, 023841 (2017)

# Semiconductor ring laser with injection

- spatially 1D along propagation direction
- active NL material
- incoherent pump
- coherent injection

## Setup




[Gustave 2015, PRL]

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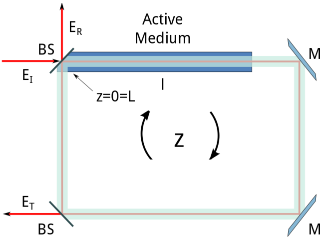


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## Model

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\begin{aligned}
F \propto \text{amplitude electric field} \Leftarrow \quad \partial_z F + \partial_t F - d\partial_z^2 F &= T[y - (1+i\theta)F + (1-i\alpha)DF] \\
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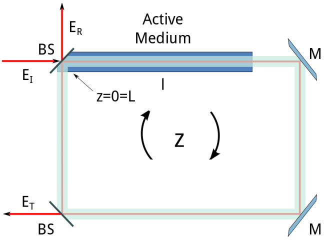


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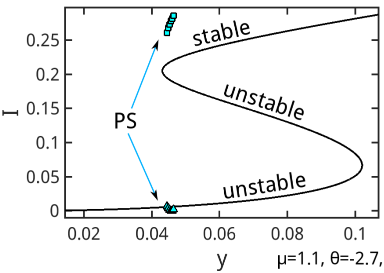


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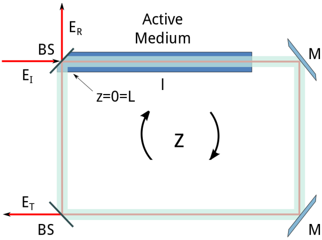


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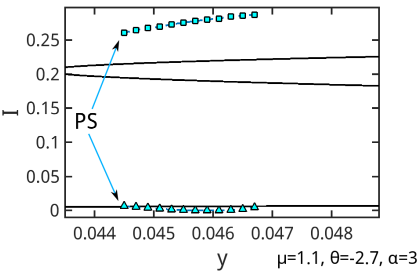
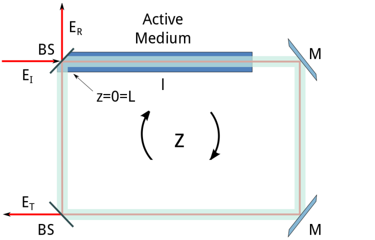



## Setup




[Gustave 2016, PRA]

## Model

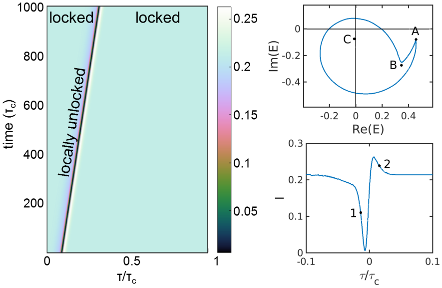
$F \propto$ amplitude electric field $\Leftarrow$ $\partial_z F + \partial_t F - \cancel{d\partial_z^2 F} = T[y - (1+i\theta)F + (1-i\alpha)DF]$

$D \propto$ amplifier carrier density $\Leftarrow$ $\partial_t D = \dfrac{bT}{\sigma}[\mu - D(1+|F|^2)]$

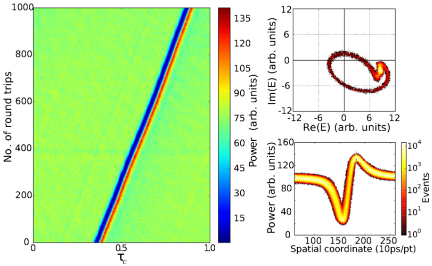
## Phase soliton

traveling localized pulse
with a positive chiral charge.

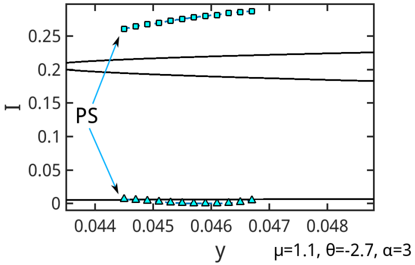
## Simulation




## Experiment




[Gustave 2015, PRL]



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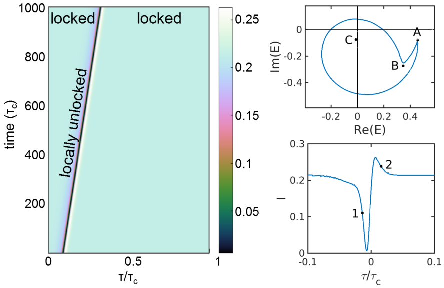
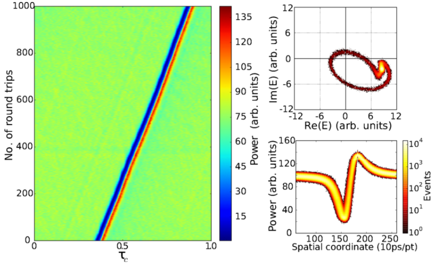



## Experiment




[Gustave 2015, PRL]

## PS generation

Phase kink
as initial condition:

$$\Phi_{+}(z) = 4\tan^{-1}[\exp(-\beta z)]$$

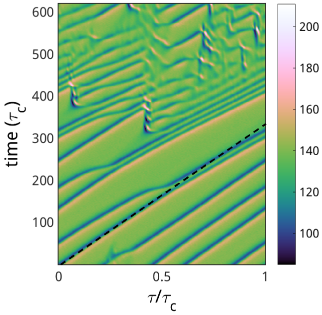
# PS complexes

## Interaction

Weakly attractive interaction experienced by PSs

$$PS_1 + PS_1 \rightarrow PS_2$$

## Experiment



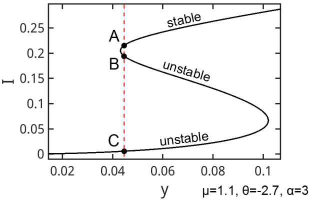

[Gustave 2017, EPJD]

# PS complexes

## Interaction

Weakly attractive interaction experienced by PSs

$$PS_1 + PS_1 \rightarrow PS_2$$



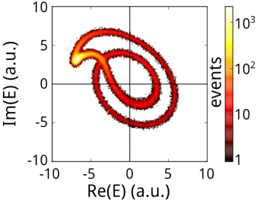

## Simulation

# PS complexes

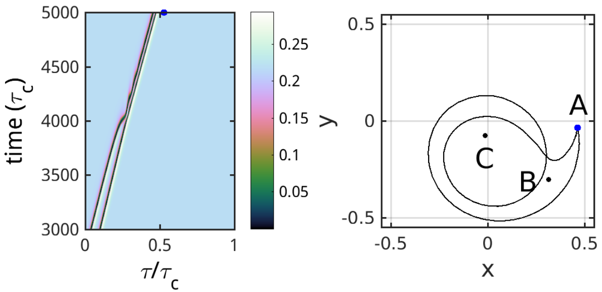
## Interaction

Weakly attractive interaction experienced by PSs

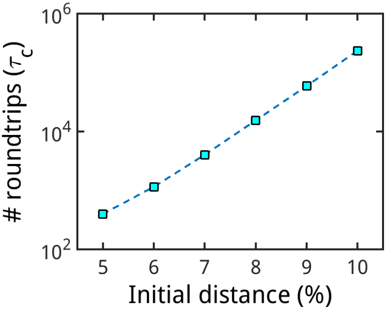
$$PS_1 + PS_1 \rightarrow PS_2$$




## Simulation

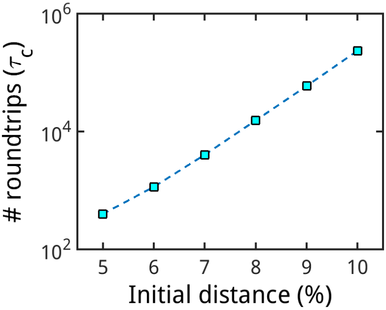


## Merging time

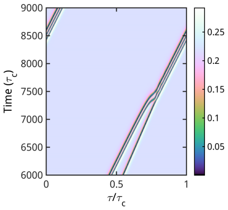



✓ **exponential** increase of **merging time** in function of the initial distance

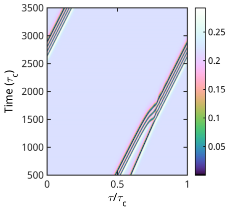
## Merging time




- ✓ **exponential** increase of **merging time** in function of the initial distance
- ✓ PS with higher charges move slower inside the cavity

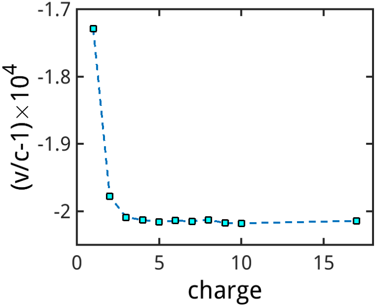



$PS_2 + PS_1 \rightarrow PS_3$

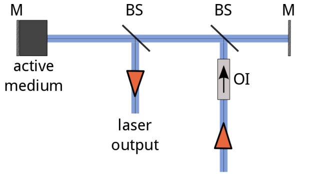



$PS_3 + PS_1 \rightarrow PS_4$

## Velocity and charge



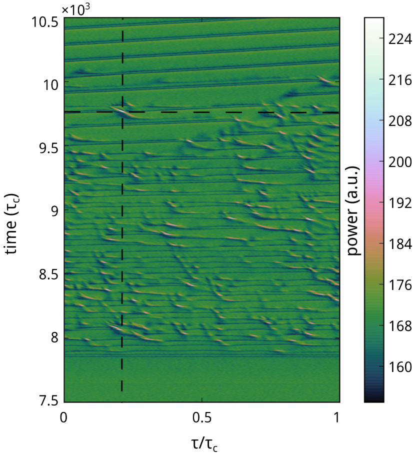
# Extreme events from collisions

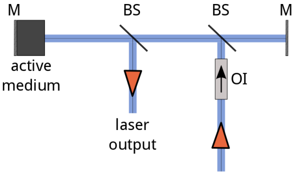



[Walczak 2017, OL]

# Extreme events from collisions

## Experiment

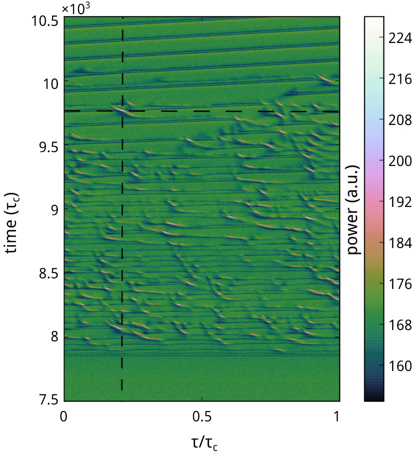


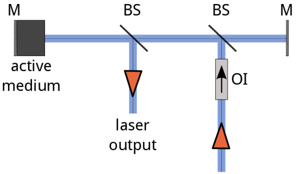




[Walczak 2017, OL]

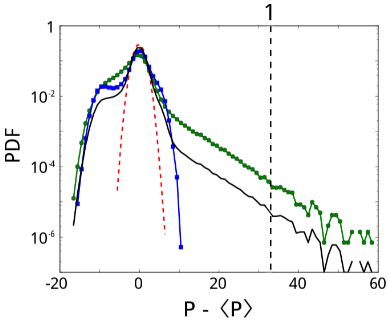
# Extreme events from collisions

## Experiment



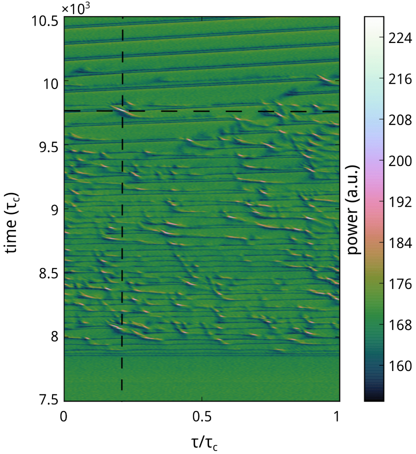




[Walczak 2017, OL]

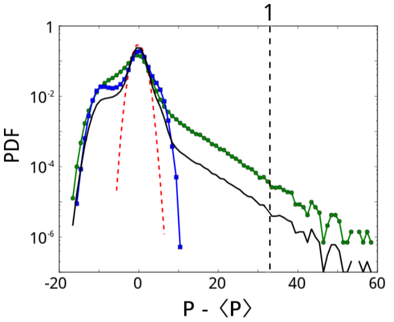


# Extreme events from collisions

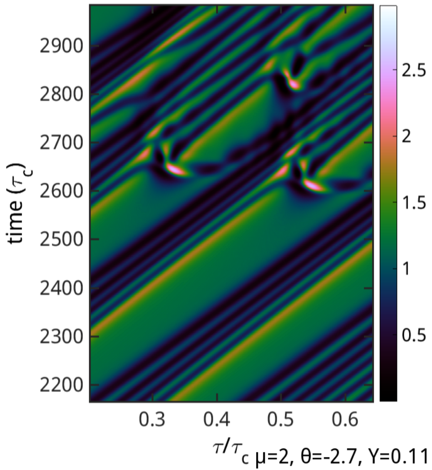
## Experiment




- ✓ Events proven extreme experimentally
- ✗ Generating mechanism?



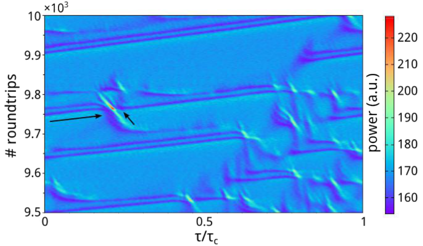
## Simulation



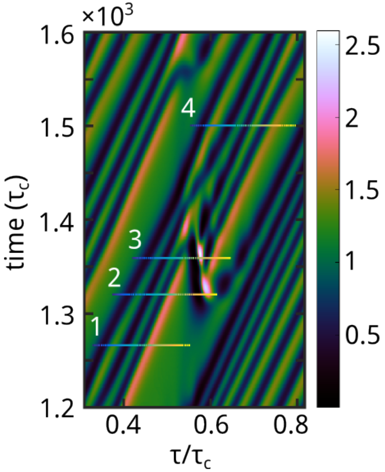

Regime for

- stable PS branch ($\theta + \alpha > 0$)
- high pump value

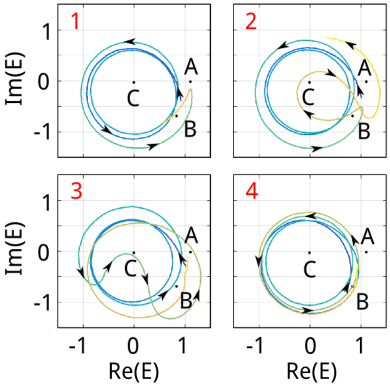
## Experiment

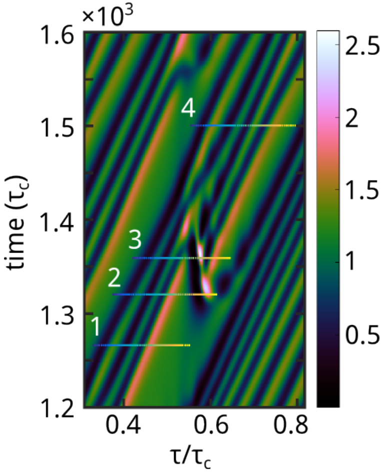



Collisions between PS complexes and other transient structures

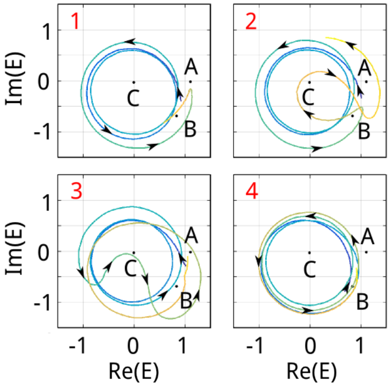

×10^3
1.6
1.5
1.4
1.3
1.2
time (τc)
0.4
0.6
0.8
τ/τc
1
2
3
4
2.5
2
1.5
1
0.5

## Phase


1
2
3
4
Im(E)
Re(E)
A
B
C
1
0
-1




## Phase

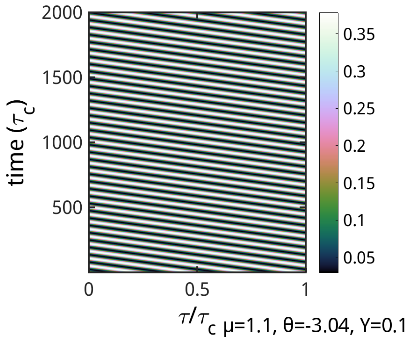



- transient structure with charge $-1$
- PS complex with charges $+n$

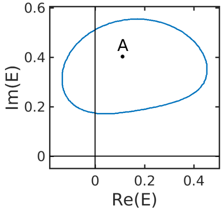
Collision with possible generation of additional positive charges

# Abnormal events in unstable roll regime

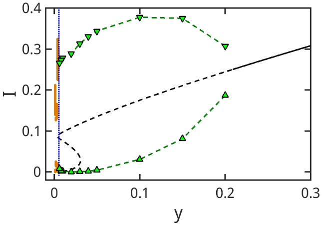
## Stable roll regime




## Argand plane



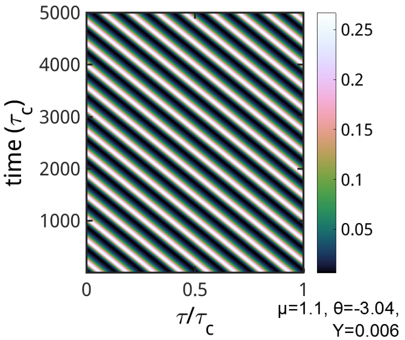

## Roll branch



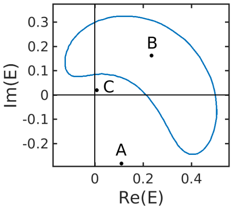

- locked HSS unstable ($\theta + \alpha < 0$)
- beating between fundamental and higher-order side-modes

# Abnormal events in unstable roll regime

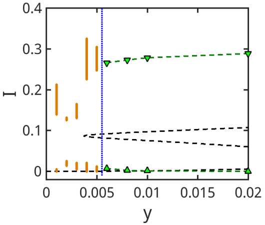
## Stable roll regime




## Argand plane

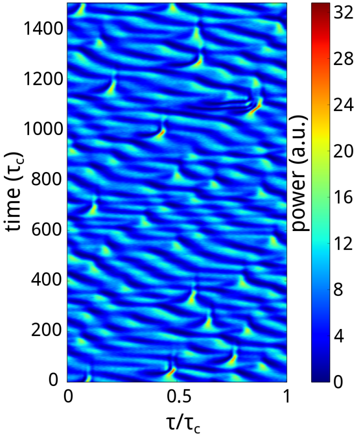



## Roll branch

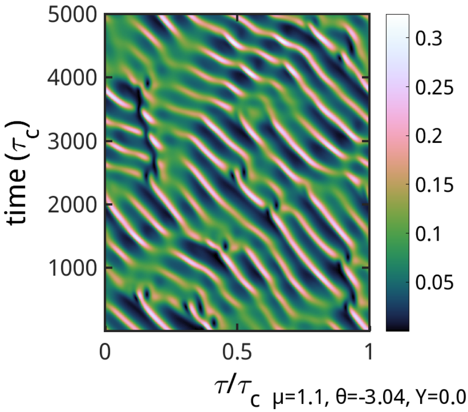



- locked HSS unstable ($\theta + \alpha < 0$)
- beating between fundamental and higher-order side-modes

## Experiment




## Simulation




μ=1.1, θ=-3.04, Y=0.004

➥ low values of injection (close to turning point)

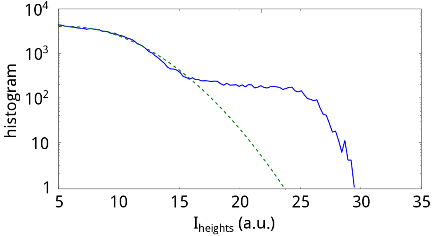
## Experiment

## Simulation

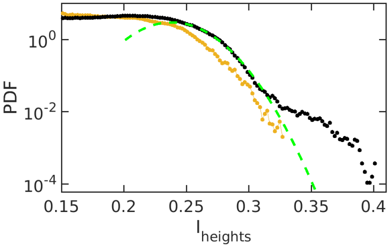
1. mostly phase bounded dynamics
2. different dynamics close to the abnormal event

## Height statistics

### Experiment

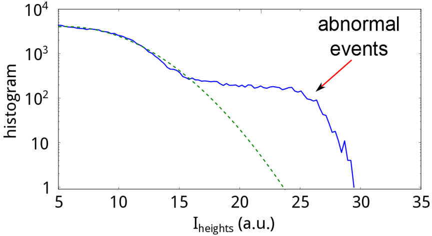



### Simulation

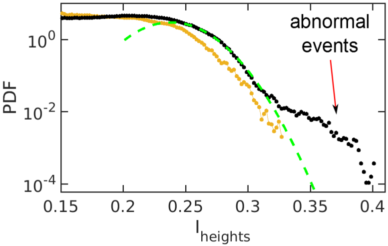


## Height statistics

**Experiment**

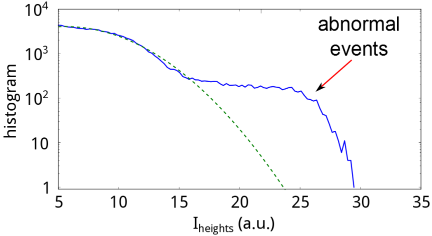



**Simulation**

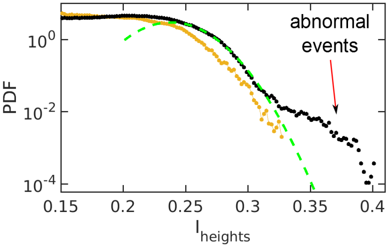


## Height statistics

### Experiment

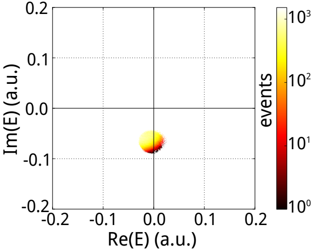



### Simulation

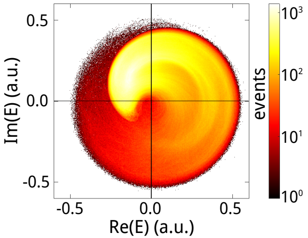


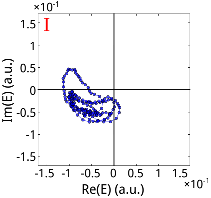

## Phase statistics

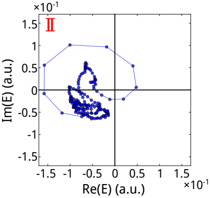
### Experiment




### Simulation

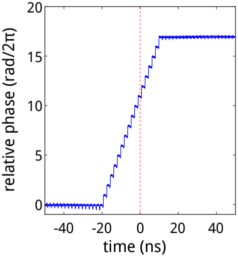




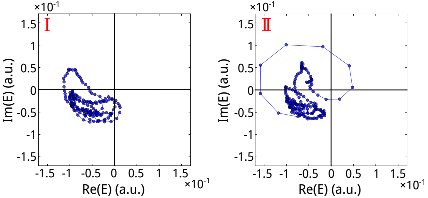




## Phase slope

Abnormal events associated with a **change in the phase slope**

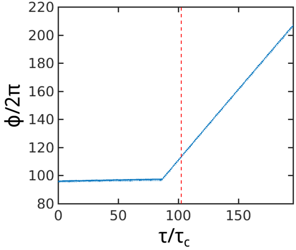



**Experiment**

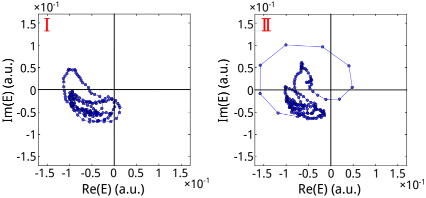



## Phase slope

Abnormal events associated with a **change in the phase slope**

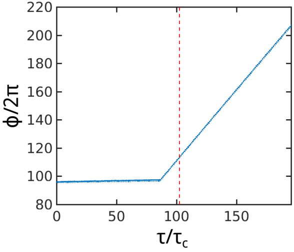



**Simulation**




## Phase slope

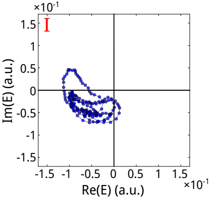
Abnormal events associated with a **change in the phase slope**

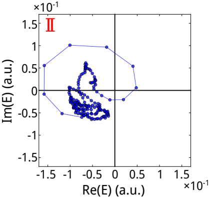



**Simulation**

## Phase equation

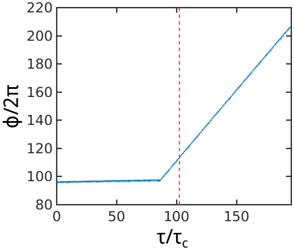
$$
\begin{aligned}
\partial_z \rho + \partial_t \rho &= T \left[ (D-1)\, \rho + y \cos\phi \right] \\
\partial_z \phi + \partial_t \phi &= -T \left[ \theta + \alpha D + \frac{y}{\rho} \sin\phi \right] \\
\partial_t D &= \frac{bT}{\sigma} \left[ \mu - D \left( 1 + \rho^2 \right) \right]
\end{aligned}
$$




## Phase slope

Abnormal events associated with a **change in the phase slope**

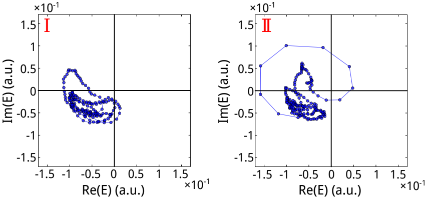



**Simulation**

## Phase equation

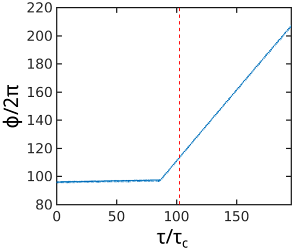
$$\partial_t \phi \ = \ -T \left[ \theta + \alpha D + \frac{y}{\rho} \sin \phi \right]$$

for fixed $z$: rotation sign determined by the sign of r.h.s.




## Phase slope

Abnormal events associated with a **change in the phase slope**



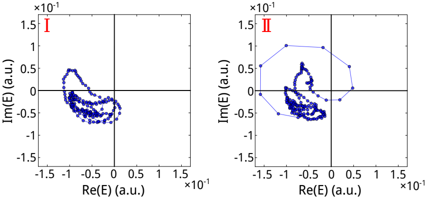

**Simulation**

## Phase equation

$$\partial_t \phi \;=\; -T\left[\theta + \alpha D + \frac{y}{\rho}\sin\phi\right]$$

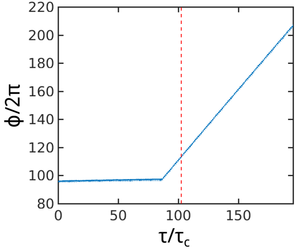
for fixed $z$: rotation sign determined by the sign of r.h.s.

➥ $y/\rho$ remains small




## Phase slope

Abnormal events associated with a **change in the phase slope**



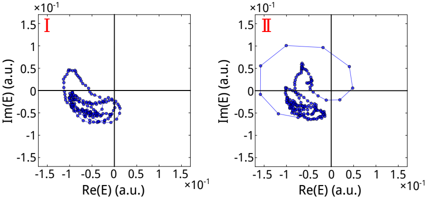

**Simulation**

## Phase equation

$$\partial_t \phi \;=\; -T\left[\theta + \alpha D + \frac{y}{\rho}\sin\phi\right]$$

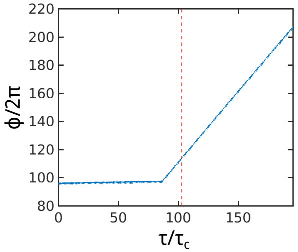
for fixed $z$: rotation sign determined by the sign of r.h.s.

- $y/\rho$ remains small
- $\theta + \alpha D$ dominant term




## Phase slope

Abnormal events associated with a **change in the phase slope**




**Simulation**

## Phase equation

$$\partial_t \phi = -T \left[ \theta + \alpha D + \frac{y}{\rho} \sin \phi \right]$$

for fixed $z$: rotation sign determined by the sign of r.h.s.

- $y/\rho$ remains small
- $\theta + \alpha D$ dominant term

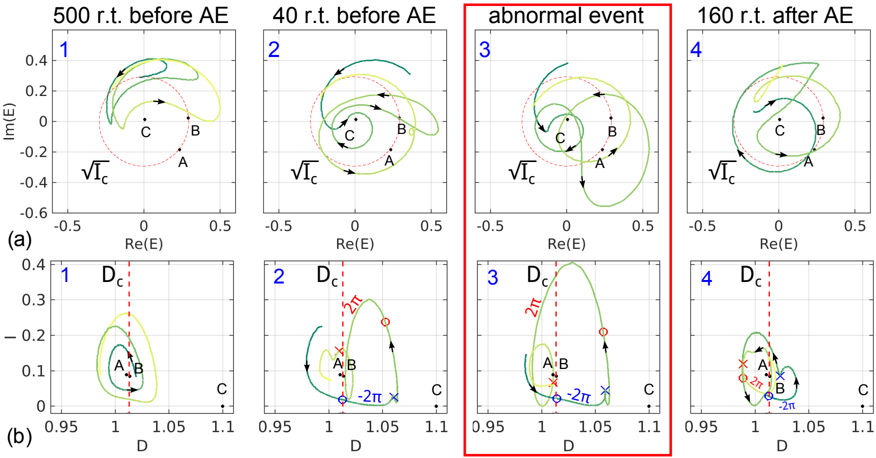
$$D_c = -\theta/\alpha$$

boundary between two rotation directions.

$D_c$ associated to $I_c$

$$-\frac{\theta}{\alpha} = \frac{\mu}{1 + I_c}$$

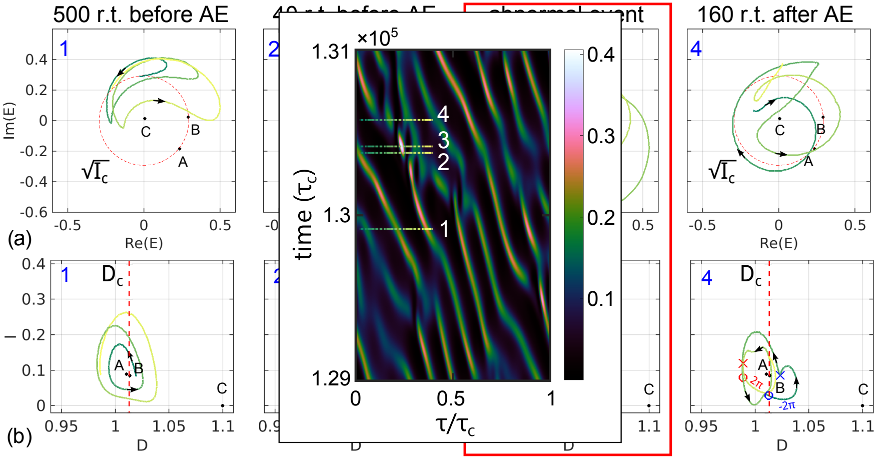
# Phase dynamics




**o** (**o**) start $2\pi$ ($-2\pi$) rotation

**x** (**x**) end $2\pi$ ($-2\pi$) rotation

# Phase dynamics




**o** (**o**) start $2\pi$ ($-2\pi$) rotation

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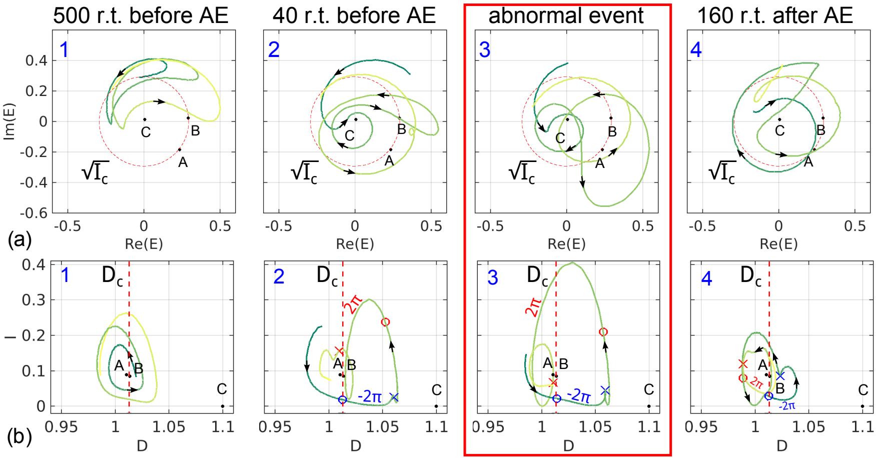
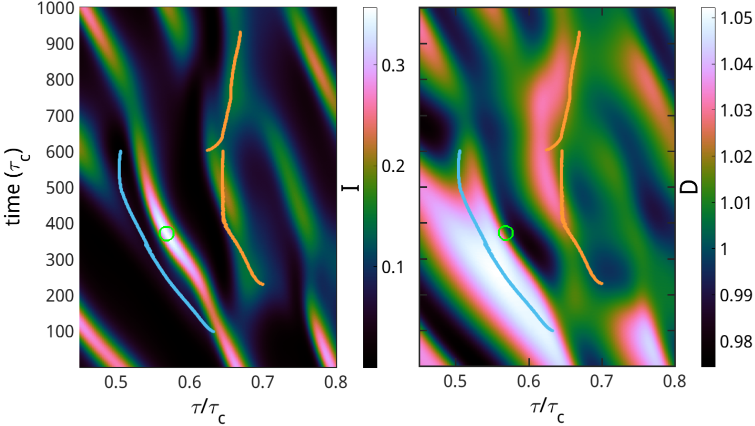
# Phase dynamics




**o** (**o**) start $2\pi$ ($-2\pi$) rotation
**x** (**x**) end $2\pi$ ($-2\pi$) rotation

**interplay of $\pm$ chiral charges**
relevant in the generation of abnormal events

# Phase dynamics



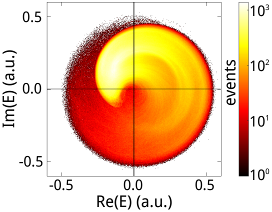

- negative charges only for high $D$ (zero intensity)
- phase slope change → loss of one of the charges

**interplay of ± chiral charges**
relevant in the generation of abnormal events

# Conclusions

## Results

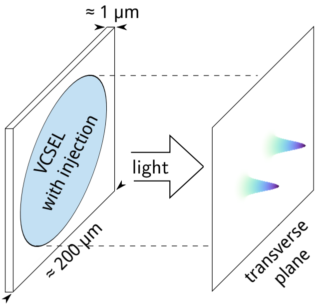
- ✓ **PS complexes** characterization and **interaction**
- ✓ **Extreme events** from **collisions** of transient structures carrying a **negative** charge and PS complexes
- ✓ **Abnormal events** emerging from **unstable roll regime** due to the interplay of $\pm$ **chiral charges**




F. Gustave, C. Rimoldi et al., *EPJD* **71**, 154 (2017)
P. Walczak, C. Rimoldi et al., *Opt. Lett.* **42**, 3000 (2017)
C. Rimoldi, F. Gustave et al., *Opt. Express* **25**, 22017 (2017)

# Broad-area semiconductor LIS

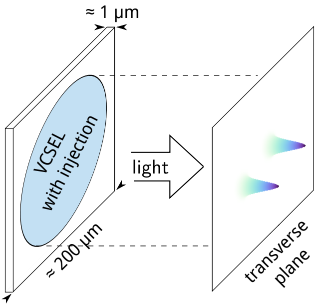
## Setup




- spatial 2D system
- active NL material
- incoherent pump
- coherent injection

# Broad-area semiconductor LIS

## Setup




- spatial 2D system
- active NL material
- incoherent pump
- coherent injection

$f(D) = (1 - \beta D)D$
for gain nonlinearity

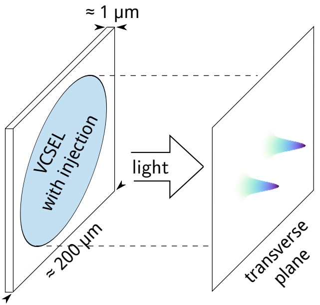
## Model

[Prati 2010, EPJD]

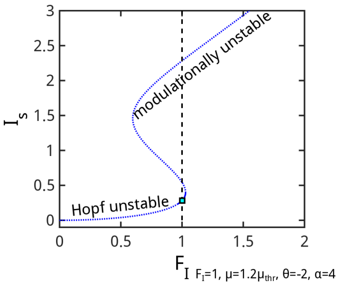
$$
\begin{aligned}
F \propto \text{amplitude electric field} \Leftarrow \quad \dot{F} &= \sigma[F_I - (1 + i\theta)F + (1 - i\alpha)f(D)F + i\nabla_\perp^2 F] \\
D \propto \text{amplifier carrier density} \Leftarrow \quad \dot{D} &= \mu - D - f(D)|F|^2 + \tilde{d}\nabla_\perp^2 D
\end{aligned}
$$

# Broad-area semiconductor LIS

## Setup




## HSS




## Model

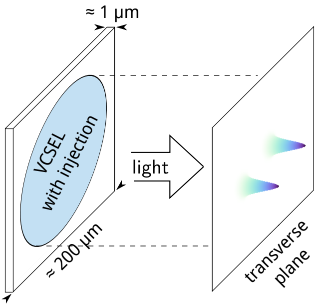
[Prati 2010, EPJD]

$F \propto$ amplitude electric field $\Leftarrow \quad \dot{F} = \sigma[F_I - (1+i\theta)F + (1-i\alpha)f(D)F + i\nabla_\perp^2 F]$

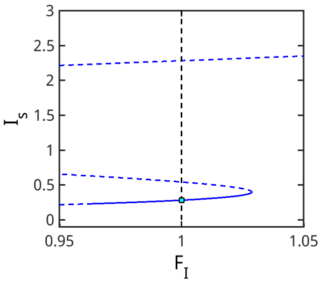
$D \propto$ amplifier carrier density $\Leftarrow \quad \dot{D} = \mu - D - f(D)|F|^2 + \tilde{d}\nabla_\perp^2 D$

# Broad-area semiconductor LIS

## Setup




## HSS




## Model

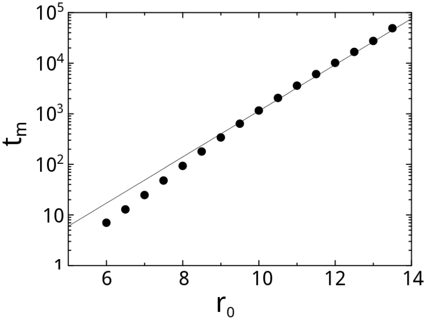
[Prati 2010, EPJD]

$$F \propto \text{amplitude electric field} \Leftarrow \quad \dot{F} = \sigma[F_I - (1+i\theta)F + (1-i\alpha)f(D)F + i\nabla_\perp^2 F]$$

$$D \propto \text{amplifier carrier density} \Leftarrow \quad \dot{D} = \mu - D - f(D)|F|^2 + \tilde{d}\nabla_\perp^2 D$$

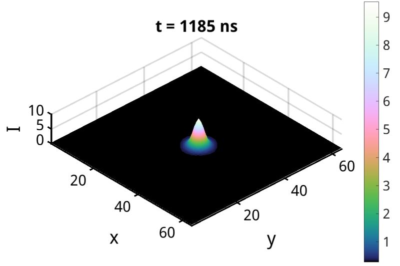
## CS interaction

## Merging time

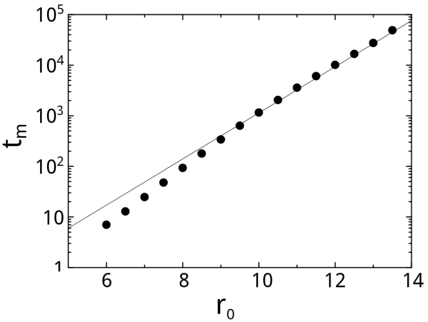



$F_I=1$, $\mu=1.2\mu_{thr}$, $\theta=-2$, $\alpha=4$, $\sigma=400$

## CS interaction




## Merging time




$F_I$=1, μ=1.2$\mu_{thr}$, θ=-2, α=4, σ=400

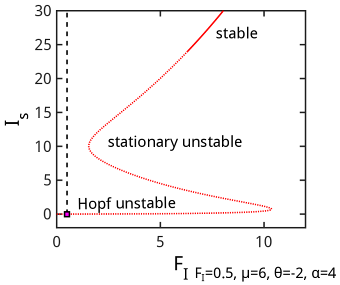
## Analytic approximation

$$\text{For } V(r) = -K^2 e^{-r/R}$$
$$\Rightarrow t_m \approx \pi \frac{R}{K} e^{r_0/(2R)}$$

- attractive force at large distances
- conservative motion of two particles under exponentially decaying potential
  $\Rightarrow$ analogy with hydrophobic materials

# Extreme event investigation

- low injection
- pump high above threshold

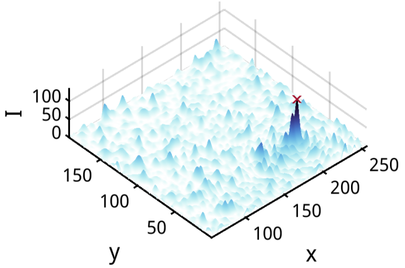



## Model

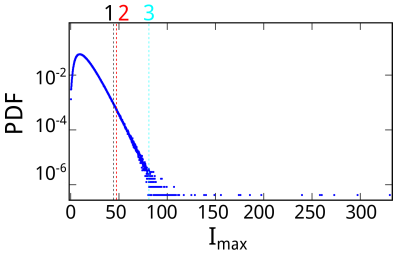
$$\dot{F} = \sigma[F_I - (1+i\theta)F + (1-i\alpha)DF + (d+i)\nabla_\perp^2 F]$$
$$\dot{D} = \mu - D(1+|F|^2)$$

$$\dot{F} = \sigma[F_I - (1+i\theta)F + (1-i\alpha)f(D)F + i\nabla_\perp^2 F]$$
$$\dot{D} = \mu - D - f(D)|F|^2 + \tilde{d}\nabla_\perp^2 D$$

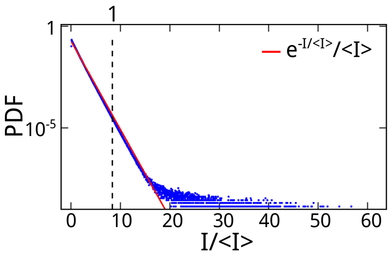
# Extreme event statistics



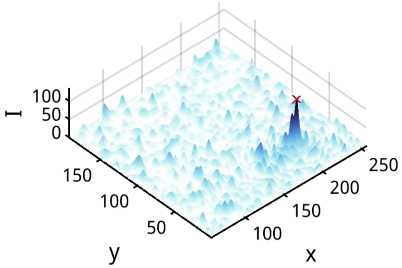

## Spatiotemporal maxima PDF



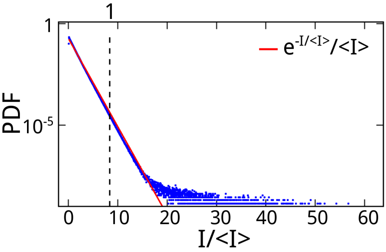

## Total intensity PDF



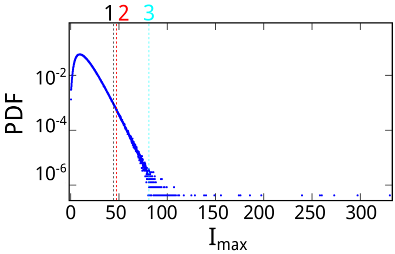
# Extreme event statistics



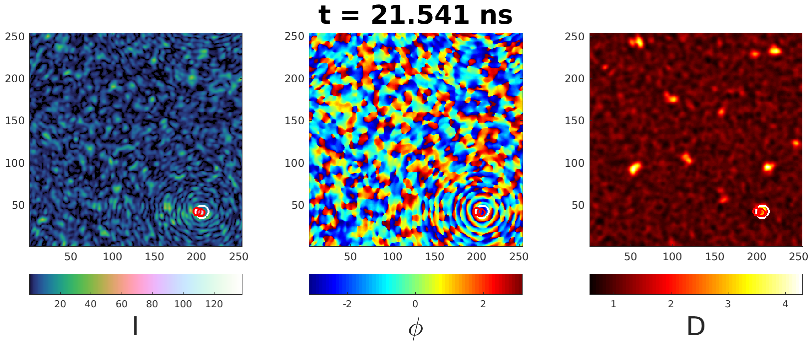

## Total intensity PDF



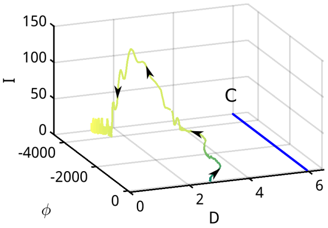

## Spatiotemporal maxima PDF

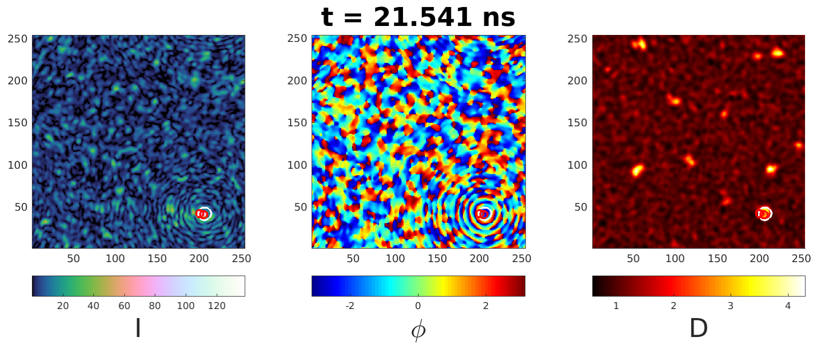



$\Rightarrow$ **Extreme events**
in the system

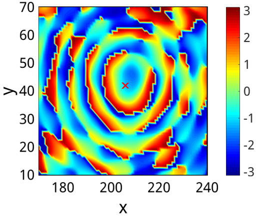

t = 21.541 ns
250
200
150
100
50
50 100 150 200 250
20 40 60 80 100 120
I
-2 0 2
$\phi$
1 2 3 4
D

## Phase space

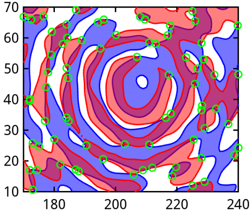

150
100
50
0
I
-4000
-2000
0
$\phi$
0 2 4 6
D
C




### Phase




### Zero isolines

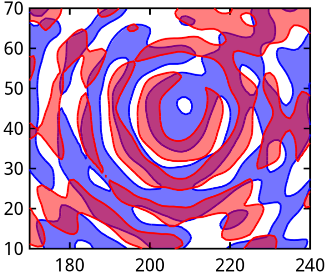



Possible connection with **vortex turbulence** [Gibson 2016, PRL] but very different dynamics.

# Conclusions

## Results

- ✓ **CS interaction** as particles under **exponentially decaying potential**
- ✓ **Extreme event** and their relation to **vortices** and **phase dynamics** (work in progress)




S. R. Anbardan, C. Rimoldi et al., *Exponentially decaying interaction potential of cavity solitons*, submitted to PRE (2017)

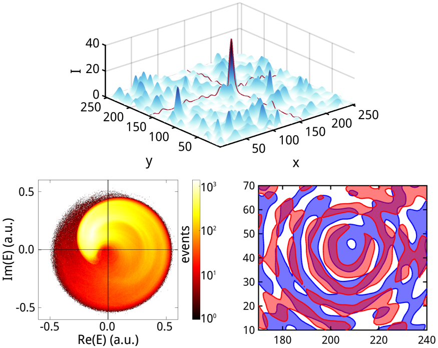
# General conclusions

- Soliton interaction
- Extreme and abnormal event formation

**Focus**

➥ Generating physical and dynamical mechanisms

1. Semiconductor laser
   - spatially 2D
     $\rightarrow$ cavity solitons
   - with saturable absorber
2. Semiconductor ring laser
   - spatially 1D (propagation)
     $\rightarrow$ phase solitons
   - with coherent injection
3. Semiconductor laser
   - spatially 2D
     $\rightarrow$ cavity solitons
   - with coherent injection



# General conclusions

- Soliton interaction
- Extreme and abnormal event formation

1. Semiconductor laser
   - spatially 2D
     → cavity solitons
   - with saturable absorber
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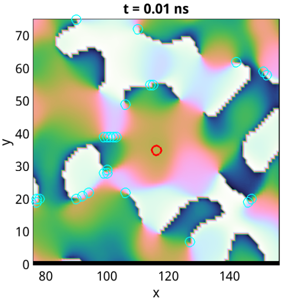
**Focus**

➥ Generating physical and dynamical mechanisms

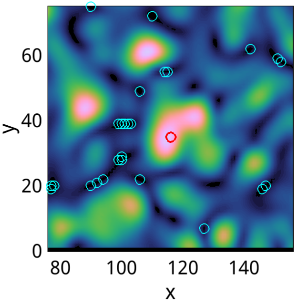
⇓

- ✎ Spatial effects
- ✎ Soliton interaction
- ✎ Modulational instability
- ✎ Chirality and vortices

# Vortices in the LSA system




$\phi(x, y)$

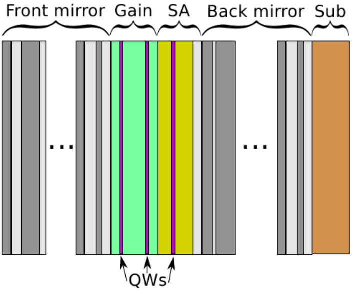



$I(x, y)$

no clear connection to extreme events

# Experimental issues LSA system

Experiments with the setup of a broad-area **monolithic** VCSEL with saturable absorber [Elsass 2010, Eur. Phys. J. D]




Courtesy of C2N/CNRS - Paris

Main issue for extreme event analysis is the need for **detectors** that are

- fast $\approx$ 1 ps
- broad-area
- spatially resolved $\approx 1\ \mu$m

# Experimental issues LSA system

Experimental solution: reduce the dimensionality of the system.
[Selmi 2016, PRL], [Coulibaly 2017, PRA]

**Experiment**

- rectangular ($\approx$ 1D) pump geometry
- pinhole before detection
- finite size of avalanche photodiode

➡ statistics on **intensity time trace averaged on a small area** $\approx 25\ \mu\text{m}^2$).

**Simulation**

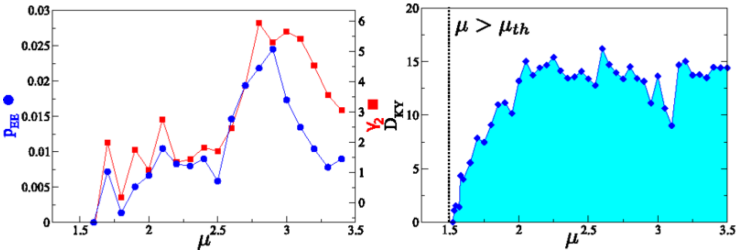
simulations run in 1D + time

➡ chaos characterization in 1D+time

➡ statistics on **spatially averaged intensity time trace** [Selmi 2016, PRL]

# Comparison with [Selmi 2016, PRL]

Comparison with **numerical results** of [Selmi 2016, PRL]



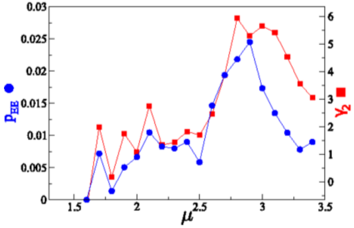

Reprinted with permission from [Selmi 2016, PRL]

- increase of the percentage of EE for higher $\mu$

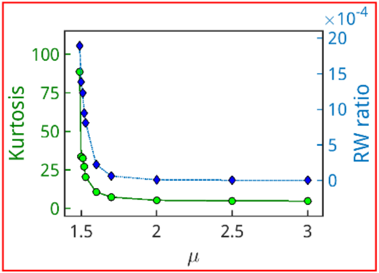
  (analysis on intensity averaged temporal trace)

- increase of the Kaplan-Yorke dimension for higher $\mu$

  (analysis on the 1D+time intensity data)

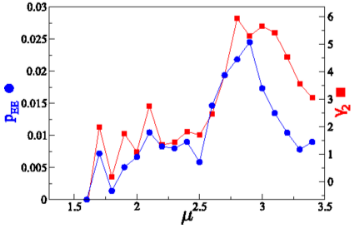



result of our analysis
on the same system




**discrepancy** due to

- 1D vs 2D geometry in the simulations
- different quantity under study (average intensity vs spatiotemporal maxima)




result of our analysis
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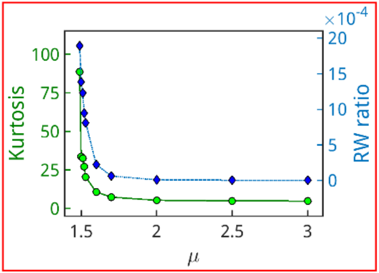
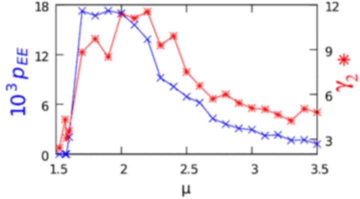


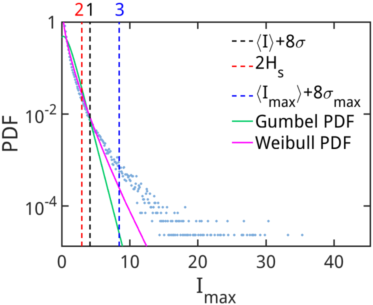




Reprinted with permission
from [Coulibaly 2017, PRA]

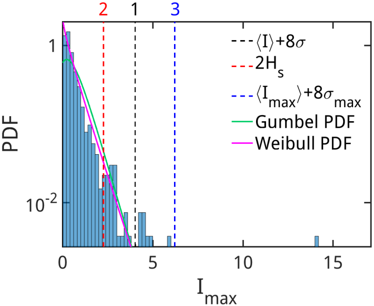
more similar results
when the authors consider
the **spatiotemporal maxima in 1D+time**

# Comparison with [Selmi 2016, PRL]

Comparison with **experimental results** of [Selmi 2016, PRL]
How does looking at the **intensity time trace averaged** on a small area affect the statistics?
**Might underestimate** the presence of EE in their most favorable regime.




**Spatiotemporal maxima PDF**

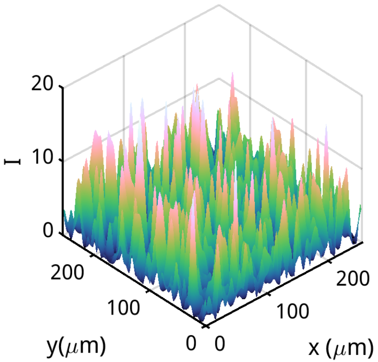



**Temporal maxima PDF**

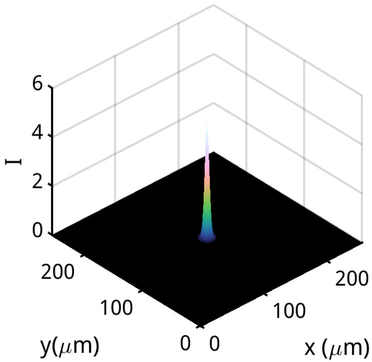
$\Rightarrow$ spatial effects
$\Rightarrow$ different quantity under study

# Fourier Spectrum

## Turbulent regime

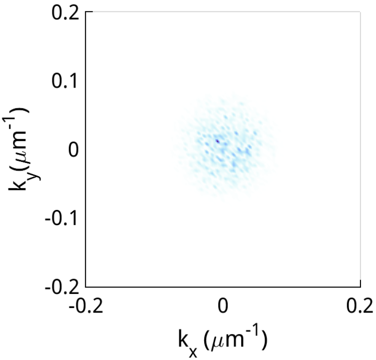



## Cavity Soliton

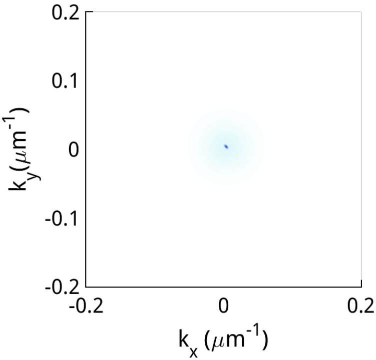


# Fourier Spectrum

## Turbulent regime

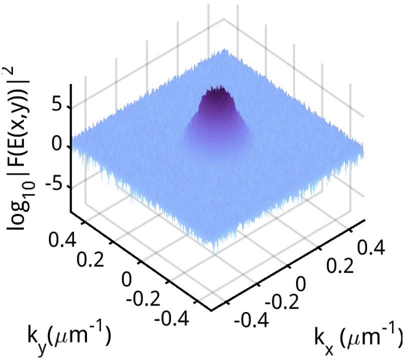



## Cavity Soliton

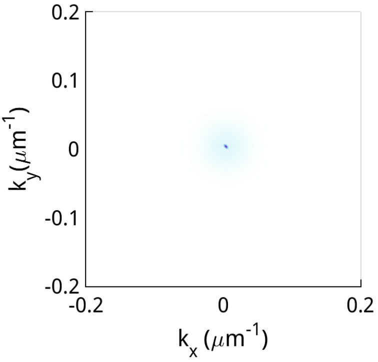


# Fourier Spectrum

## Turbulent regime

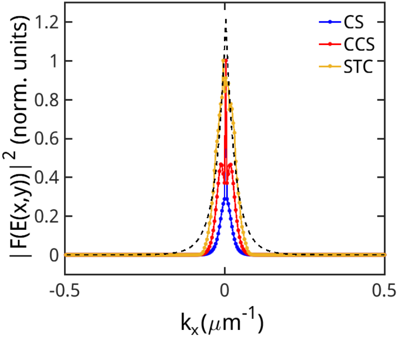



## Cavity Soliton

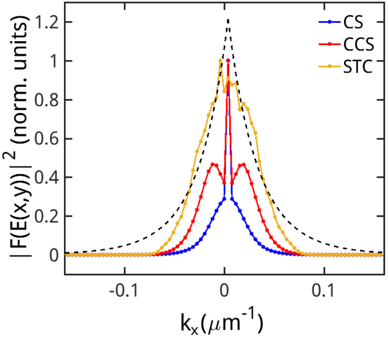


# Fourier spectrum LSA

## Fourier spectrum

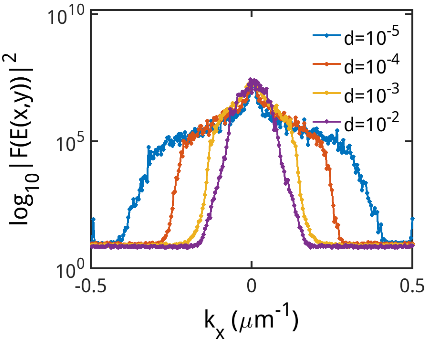



## Zoom of the spectrum



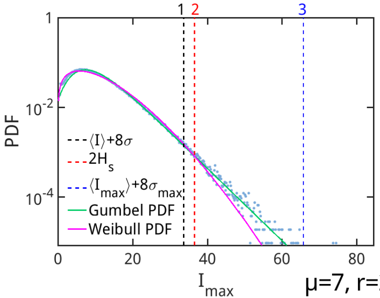

spectrum broadening in the spatiotemporal chaos regime

# Effect of the diffusion coefficient in LSA




- **filter** on spatial frequencies in order to **avoid filamentation** due to high spatial frequency excitation.
- **proven theoretically** in [Fedorov 2000, PRE]
- Specific **value** here **chosen phenomenologically** (**smallest** value **to avoid self-collapsing** without disrupt or change the dynamics of the model and its solutions).

# Weibull and Gumbel distributions




μ=7, r=2.5

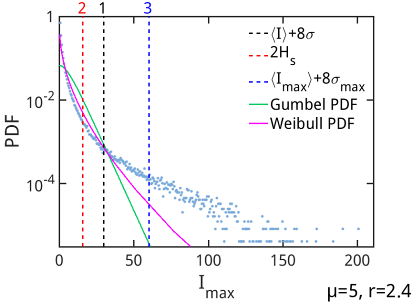
$$Wei_{PDF}(I_{max}) = \frac{k}{\lambda}\left(\frac{I_{max}}{\lambda}\right)^{k-1}\exp\left[-\left(\frac{I_{max}}{\lambda}\right)^{k}\right]$$

$$Gum_{PDF}(z) = \frac{1}{\beta}\exp[-z - \exp(-z)]$$

with $z = \frac{I_{max} - \langle I_{max} \rangle}{\beta} + \gamma$
from Extreme Value Theory

Study of the limit distribution for the maxima of sequences of indipendent and identically distributed variables

# Weibull and Gumbel distributions



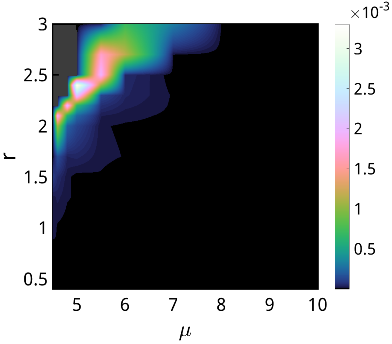

$$Wei_{PDF}(I_{max}) = \frac{k}{\lambda}\left(\frac{I_{max}}{\lambda}\right)^{k-1} \exp\left[-\left(\frac{I_{max}}{\lambda}\right)^{k}\right]$$

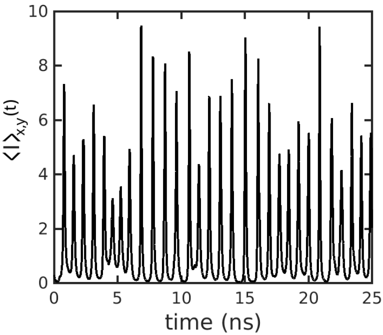
$$Gum_{PDF}(z) = \frac{1}{\beta}\exp[-z-\exp(-z)]$$

with $z = \frac{I_{max} - \langle I_{max}\rangle}{\beta} + \gamma$
from Extreme Value Theory

Study of the limit distribution for the maxima of sequences of indipendent and identically distributed variables

# Favorable regime for EE

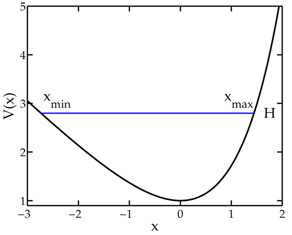


# Conservative limit LSA

1D case

$$\dot{F} = \left[(1-i\alpha)D + (1-i\beta)\bar{D} - 1\right] F$$
$$\dot{D} = b\left[\mu - D\left(1+|F|^2\right)\right]$$
$$\dot{\bar{D}} = rb\left[-\gamma - \bar{D}\left(1+s|F|^2\right)\right]$$




[Oppo 1985], [Cialdi 2013]
for class-B laser

$$x = \log\frac{I}{I_s},\ D = D_s(1+n),\ \bar{D} = \bar{D}_s(1+\bar{n})$$

$$\ddot{x} + 2b\left(D_s + s\bar{D}_s r\right) I_s(e^x - 1)$$
$$=$$
$$-2b\left(D_s n + \bar{D}_s r\bar{n}\right) - 2b\left(D_s n + \bar{D}_s rs\bar{n}\right) I_s e^x$$

Conservative limit:

$$\ddot{x} + \frac{dV_{LSA}(x)}{dx} = 0$$

motion of a unitary mass oscillator

$$\omega^2_{LSA} = 2b\left(D_s + s\bar{D}_s r\right) I_s$$

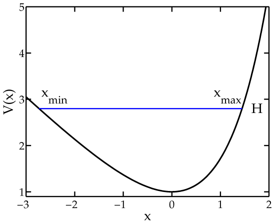
$$V_{LSA}(x) = \omega^2_{LSA} V(x)$$

$V(x) = e^x - x$ Toda potential

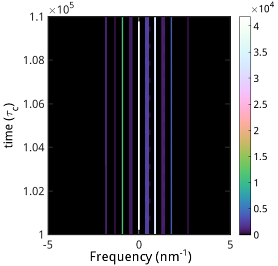
# Conservative limit LSA 2D+time

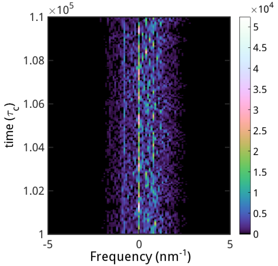
$$
\begin{aligned}
\dot{F} &= F\left[D(1-i\alpha)+\bar{D}(1-i\beta)-1+(\delta+i)\nabla_\perp^2\right] \\
\dot{D} &= -bD_s\left(I-I_s\right) \\
\dot{\bar{D}} &= -rb\bar{D}_s s\left(I-I_s\right)
\end{aligned}
$$

Not yet tested numerically

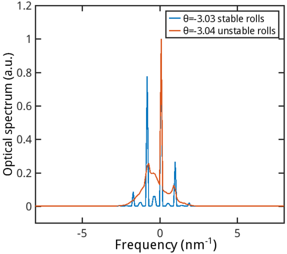


# Fourier spectrum ring laser with injection




stable roll regime ($\theta = -3.03$) unstable roll regime ($\theta = -3.04$)



# Chirality

## Phase solitons

Phase solitons carry only **positive** charges

## Symmetry breaking

- propagative nature of the system
- presence of $D$ whose dynamics develops on a different temporal scale

# Modified forced Ginzburg-Landau model

$$\left[1+2i\tilde{d}\sigma\delta(1)\right]\frac{\partial E}{\partial \eta}+\frac{\partial E}{\partial \tau}-\frac{\tilde{d}\sigma^2}{T}\frac{\partial^2}{\partial \eta^2}E$$
$$= \quad T\left\{y-\left[1-\mu+i\left(\mu\alpha+\theta\right)\right]E-(1-i\alpha)|E|^2E\right\} \ ,$$

with

$$\tilde{d}=\frac{1+i\alpha}{\Gamma(1)^2(1+\alpha^2)}$$

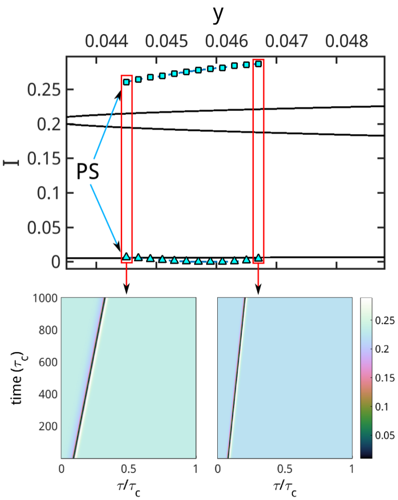
term of diffusion + dispersion since, at order $\epsilon=\mu-1$

$$\frac{\partial E}{\partial \tau'}=-\eta_0\frac{\partial E}{\partial \eta} \ ,$$

- dispersion can be present
- usually it is small, hence neglectable in the PS description and model dynamics

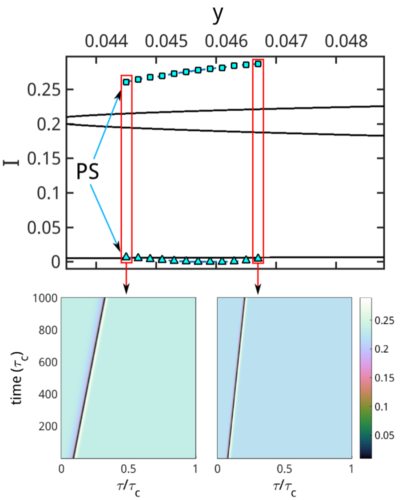
# Single-charge PS

## PS velocity

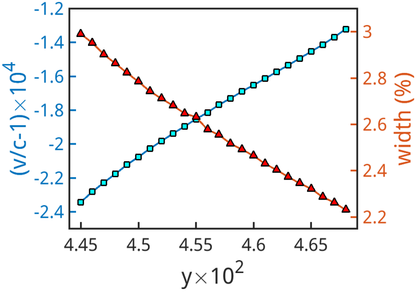


# Single-charge PS

## PS velocity

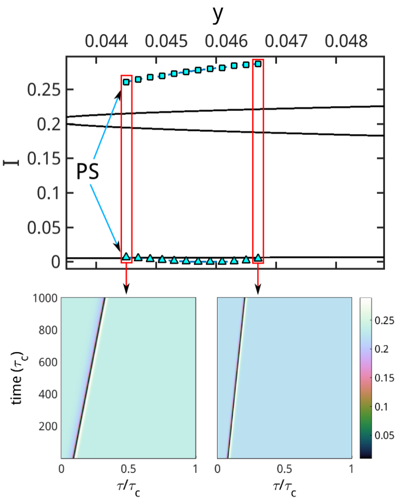



## Velocity and size

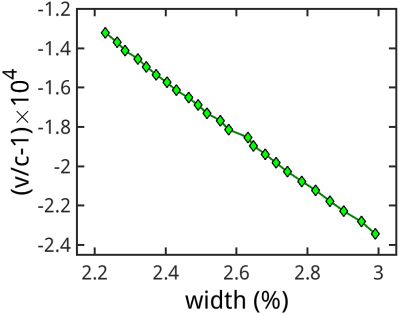


# Single-charge PS

## PS velocity

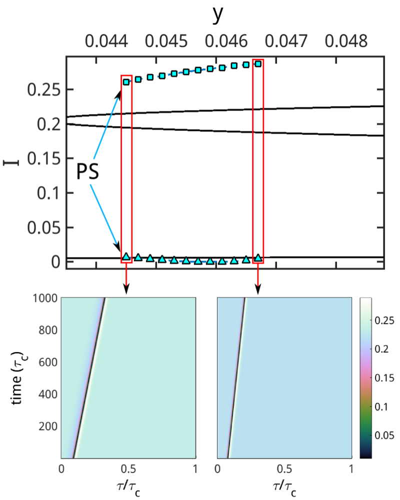



## Velocity and size

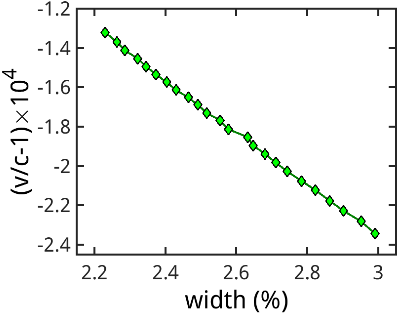


# Single-charge PS

## PS velocity



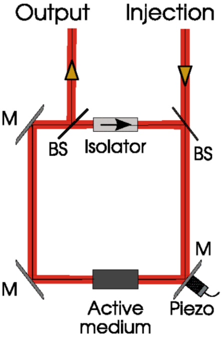

## Velocity and size

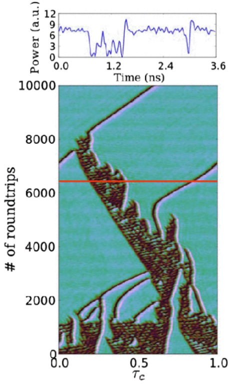


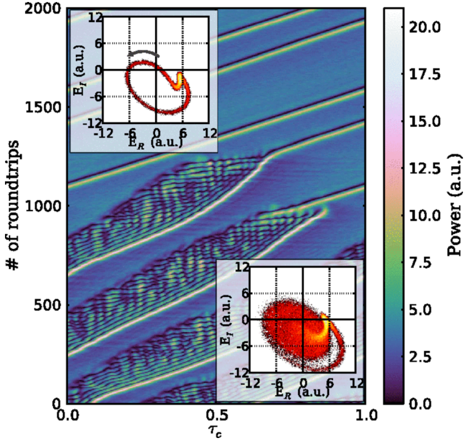

For increasing values of injection

- ✓ linear increase of PS velocity
- ✓ linear decrease of PS size

# Front propagation in ring laser



# Analogy with hydrophobic materials

Interaction of CSs as particles subject to an **exponentially decaying potential**

$\Downarrow$

Potential often associated to the **hydrophobic force**

$\rightarrow$ force experienced by nonpolar molecules and surfaces in water

[Israelachvili 1982, Nature]
[Donaldson 2015, Langmuir]