Vortex Reconnections and Density Waves in Trapped Bose-Einstein Condensates

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Physical Review X **7**, 021031 (2017)

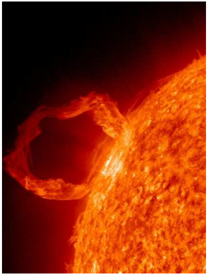
SIG on Wave Turbulence

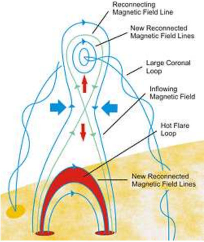
Birmingham, 11 December 2017

Overview

Overview

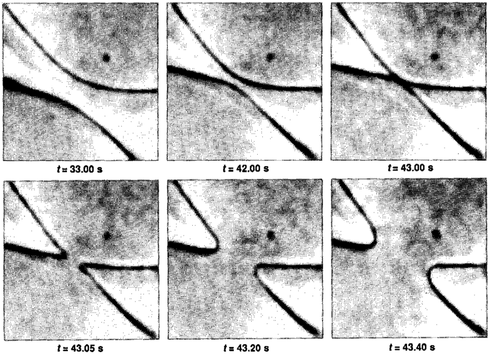
Plasmas





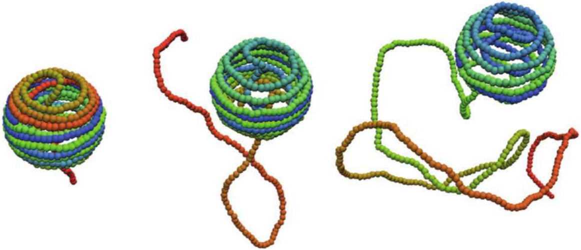
- reconnections of magnetic field lines
  [Zhike *et al.*, *Nat. Com.* (2016)]
- anomalous heating of solar corona
  [Cirtain *et al.*, *Nature* (2013)]
- explosive events, solar and stellar flares
  [Che *et al.*, *Nature* (2011)]

Nematic Liquid Crystals



[Chuang *et al.*, *Science* (1991)]

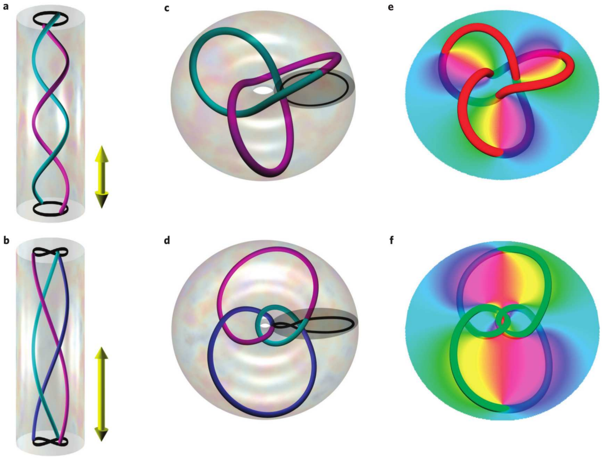
Polymers and DNA



[Marenduzzo *et al.*, *PNAS* (2009)]

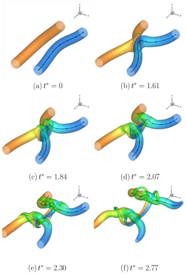
M. Vazquez (UC DAVIS) De W. Sumners (Florida)

Optical beams

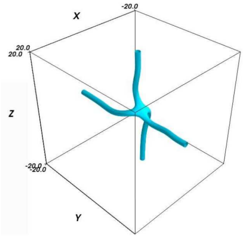


[Dennis *et al.*, *Nature* (2010)]

Classical and Quantum Fluids



[Hussain *et al.*, *Phys. Fluids* (2011)]

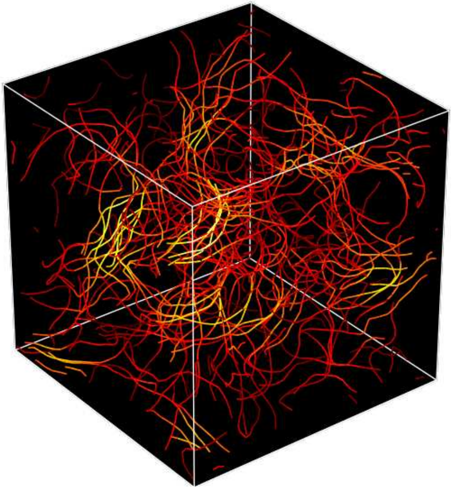


[Zuccher *et al.*, *Phys. Fluids* (2012)]

Reconnections

- trigger turbulent energy cascade
- redistribute helicity among scales
- enhance fine–scale turbulent mixing

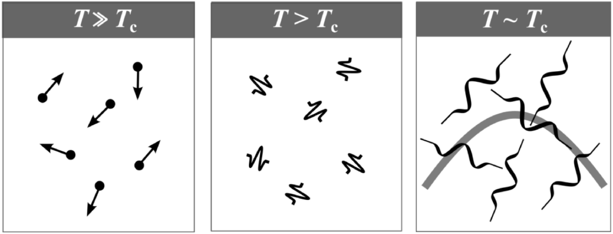
Quantum Fluids



[Baggaley *et al.*, *Phys. Rev. Lett.*, **109**, 205304 (2012)]

- neutron stars
- ^{4}He
- ^{3}He-B
- BECs

Bose-Einstein Condensation



- $\lambda_T = \dfrac{h}{\sqrt{2\pi m k_B T}}$ *vs* $d \sim \left(\dfrac{V}{N}\right)^{1/3} = n^{-1/3}$
- $\lambda_T^3 n \ll 1$ *classical gas* Maxwell-Boltzmann **p** distribution
- $\lambda_T^3 n \gtrsim 1$ *quantum gas*
  - Fermi-Dirac distribution *FERMIONS*
  - Bose distribution *BOSONS*
- bosons , $T < T_c$ $N_0(T) = N[1-(T/T_c)^{3/2}]$ *ideal* Bose Gas
- macroscopic $\Psi_0(\mathbf{x},t) = \sqrt{n_0(\mathbf{x},t)}\,e^{i\theta(\mathbf{x},t)}$, $||\Psi_0||^2 = N$

Weakly interacting Bose-Einstein Condensates (BECs)

- Gross-Pitaevskii model $a_s \ll n^{-1/3}$

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V(\boldsymbol{x},t)\Psi + g|\Psi|^2\Psi, \qquad g = \frac{4\pi\hbar^2 a_s}{m}$$

- Madelung transformation $\Psi = \sqrt{n}e^{i\theta}$, $\mathbf{v} = \frac{\hbar}{m}\nabla\theta$

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = 0$$

$$mn\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right] = -\nabla\left(p + p_Q\right) - n\nabla V$$

- $p = \frac{gn^2}{2}$, $p_Q = -\frac{\hbar^2}{4m}n\nabla^2(\ln n)$ quantum pressure
- $\Delta \gg \xi = (\hbar^2/2mgn)^{1/2}$ recover compressible Euler
- $p_Q \Rightarrow$ reconnections

Classical *vs* Quantum fluids

- Classical Euler (inviscid) fluids

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\,\mathbf{v}\right]=-\nabla p$$

NO reconnections, $\dot{E}=0$, $\mathscr{H}$ constant

- Classical Navier-Stokes (viscous) fluids

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\,\mathbf{v}\right]=-\nabla p+\eta\nabla^2\mathbf{v}$$

reconnections driven by dissipation, $\dot{E}<0$, $\mathscr{H}$?

- Quantum BECs

$$mn\left[\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\,\mathbf{v}\right]=-\nabla p-\nabla p_Q$$

reconnections driven by quantum pressure, $\dot{E}=0$, $\mathscr{H}$?

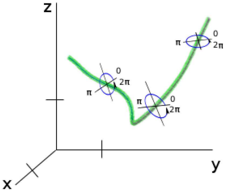
Classical Vortices

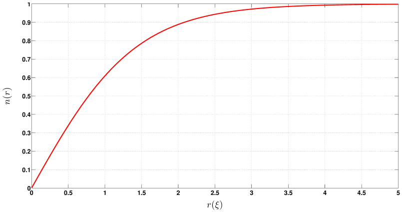
In classical viscous fluids vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{v}$ is unconstrained

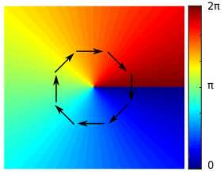
- vortices any shape, size, orientation
- vortex vorticity flux (circulation Γ) any strength
- vortex core is finite, arbitrary

Quantized Vortices

- rotation, temperature quench
- one–dimensional structures
- topological defects of order parameter Ψ
- $\boldsymbol{\omega}_s$ confined to vortex lines, $\boldsymbol{\omega}_s = \kappa \oint_{\mathscr{L}} \mathbf{s}'_i(\zeta, t)\delta^{(3)}(\mathbf{x} - \mathbf{s}_i(\zeta, t))\, d\zeta$
- $\kappa = n\dfrac{h}{m}$, $n \in \mathbb{Z}$, quantized $\mathbf{v} = \dfrac{h}{2\pi m}\nabla\theta$





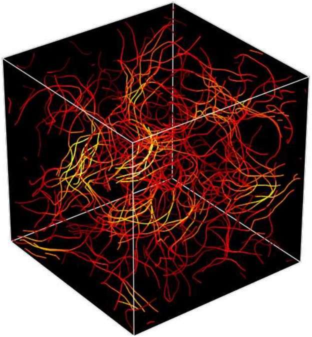


RECONNECTIONS EASIER TO STUDY

Overview

Importance of Quantum Vortex Reconnections

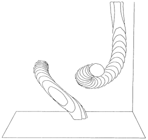
- redistribute energy among scales
- redistribute helicity among scales
  [Scheeler *et al.*, *PNAS* (2014)]
- enhance mixing
- trigger a Kelvin Wave cascade
  [Kivotides *et al.*, *Phys. Rev. Lett.* (2001)]
- transform incompressible kinetic energy into acoustic energy
  [Leadbeater *et al.*, *Phys. Rev. Lett.* (2001)]
- in superfluid ^{4}He turbulence
  - Kolmogorov spectrum



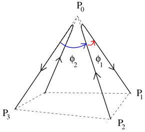
[Baggaley *et al.* *Phys. Rev. Lett.* (2012)]

Homogeneous superfluid systems

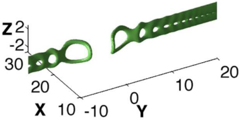
- confirmation the Feynman conjecture
  [Koplik & Levine, *Phys. Rev. Lett.* (1993)]



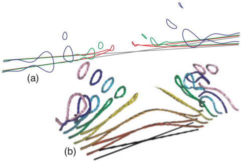
- *universal route* to reconnection
  via formation pyramidal cusp
  [de Waele & Aarts, *Phys. Rev. Lett.* (1994)]
  [Tebbs *et al.*, *J. Low Temp. Phys.*, (2011)]



- induced cascade of vortex rings



[Kerr, *Phys Rev Lett*(2011)]



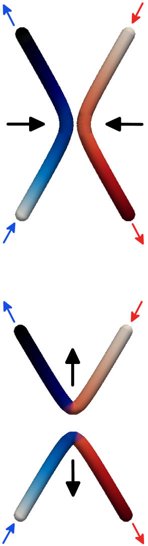
[Kursa *et al.*, *Phys Rev B* (2011)]

Homogeneous superfluid systems

- scaling $\delta(t) \sim t^{\alpha}$
- conservation of helicity (*knottedness*)

$$\mathscr{H} = \int_V \mathbf{v} \cdot \boldsymbol{\omega} \, d\mathbf{x}$$

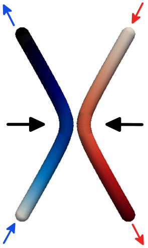
- decay of knots *via* particular pathways



Homogeneous superfluid systems

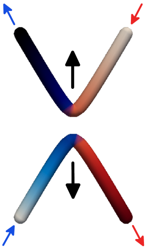
- scaling $\delta(t) \sim t^{\alpha}$

  [de Waele & Aarts, *Phys. Rev. Lett.*, **72**, 482 (1994)]
  [Nazarenko & West, *J. Low. Temp. Phys.*, **132**, 1 (2003)]
  [Bewley *et al.* *PNAS*, **105**, 13707 (2008)]
  [Tebbs *et al.*, *J. Low. Temp. Phys.*, **162**, 314 (2011)]
  [Zuccher *et al.*, *Fluids. Phys*, **24**, 125108 (2012)]
  [Villois *et al.*, *Phys. Rev. Fluids*, **2**, 044701 (2017)]



- conservation of helicity $\mathcal{H}$ (*knottedness*)

  [Scheeler *et al.*, *PNAS*, **111**, 15350 (2014)]
  [Laing *et al.*, *Sci. Rep.*, **5**, 9224 (2015)]
  [Zuccher & Ricca *Phys. Rev. E*, **92**, 06101 (2015)]
  [Di Leoni *et al.*, *Phys. Rev. A*, **94**, 043605 (2016)]
  [Salman, *Proc. R. Soc. A*, **473**, (2017)]

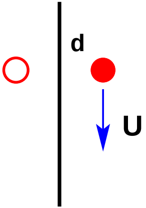


- decay of knots *via* particular pathways

  [Kleckner *et al.*, *Nat. Phys.*, **12**, 650 (2016)]

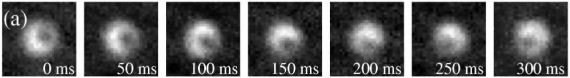
Inhomogeneous BECs

- boundaries, image vortices



- semi-infinite BEC
- $U \approx \dfrac{\hbar}{2m(d-\sqrt{2}\xi)}$
- [Mason *et al.*, *Phys. Rev. A* **74**, 021701 (2006)]

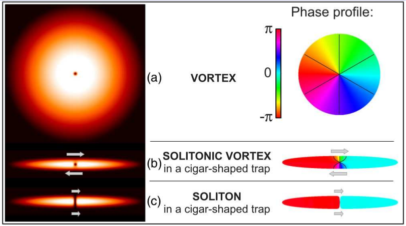
- non-homogeneous $V(\mathbf{x},t)$, $\nabla\rho$ drive vortex motion



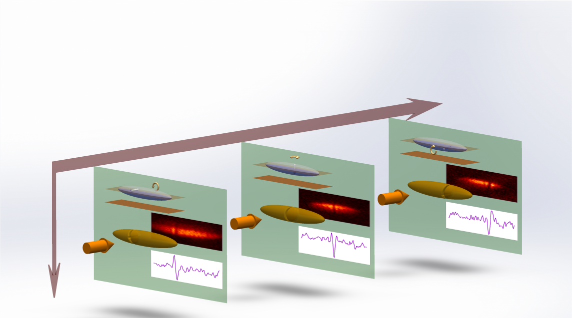
[Anderson *et al.*, *Phys. Rev. Lett.* **85**, 2857 (2000)]

Trento Experiments
Trapped 3D BEC

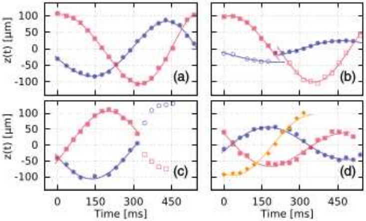
- cigar–shaped BEC of Na atoms
- Kibble-Zurek (quench) generation of solitonic vortices [Lamporesi *et al.*, *Nat. Phys.* (2013)]
- real time imaging of $\Delta N/N \simeq 4\%$

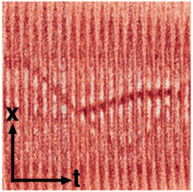


[Donadello *et al.*, *Phys. Rev. Lett.* (2014)]



Vortex Interactions
Real Time Imaging





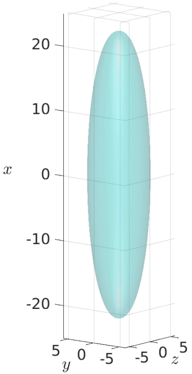
[Serafini *et al.*, *Phys. Rev. Lett.* (2015)]

1st direct evidence vortex interactions 3D BECs!

Overview

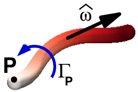
Gross–Pitaevskii Model, $T = 0$

$$i\frac{\partial \Psi}{\partial t} = -\frac{1}{2}\nabla^2\Psi + \frac{1}{2}\left[\left(\frac{\omega_x}{\omega_\perp}\right)^2 x^2 + y^2 + z^2 + \right]\Psi + \frac{4\pi N a_s}{\ell}|\Psi|^2\Psi$$



VORTEX TRACKING

- Pseudo–vorticity field $\hat{\boldsymbol{\omega}}$
- $\Psi = \sqrt{n}e^{i\theta} = 0, \quad d\Psi = 0$
- $\hat{\boldsymbol{\omega}} = \dfrac{\nabla n \times \nabla \theta}{|\nabla n \times \nabla \theta|}$

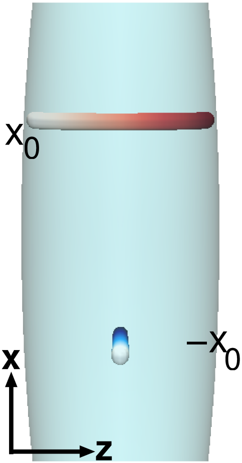


[Rorai *et al.*, *J. Fluid Mech.* (2016)]

[Villois *et al.*, *J. Phys. A* (2016)]

Overview

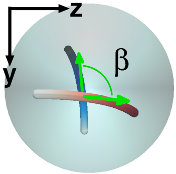
Vortex Dynamics
Initial vortex configurations



- $\beta_0 = \pi/2$
- equidistant from radial plane $x = 0$

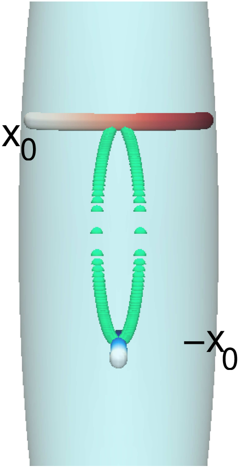
Focus

- $x_G(t)$
- $\delta(t)$
- $\beta(t)$

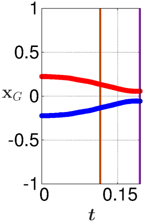


Vortex Dynamics

Stage I : 2 single-vortex motions



- $\delta \geq R_\perp$
- elliptical
- $\chi = x_0 / R_x = r_{max} / R_\perp$
- isolines $V(\mathbf{x})$
- $v \propto \nabla\rho/\rho \propto \chi/(1-\chi^2)$
- outer vortices faster



$$T = \frac{8\pi(1-\chi^2)\mu}{3\hbar\omega_\perp\omega_z \ln(R_\perp/\xi)}$$

[Lundh, Ao, PRA, **61** (2000)]

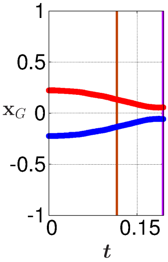
[Svidzinsky, Fetter, PRL, **84** (2000)]

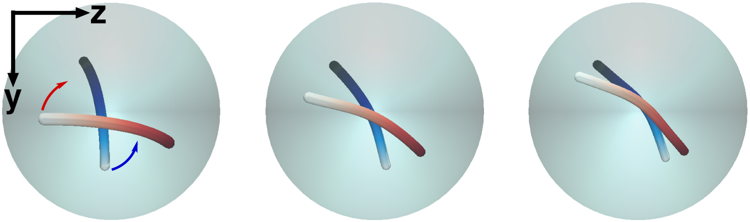
[Sheehy, Radzihovsky, PRA, **70** (2004)]

Vortex Dynamics

Stage II : Rotation on radial plane

- $\delta \sim R_\perp$
- anti-parallel configuration
- slows down axial motion

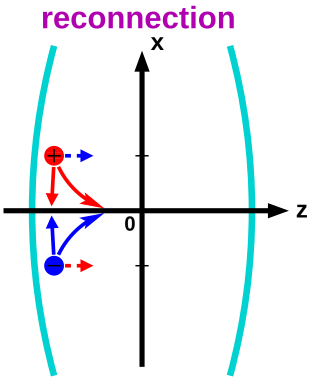


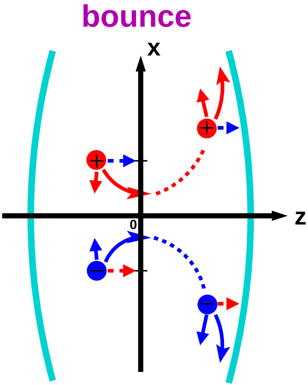


t_1 t_2 t_3

Vortex Dynamics

Stage III : Two COMPETING Dynamics



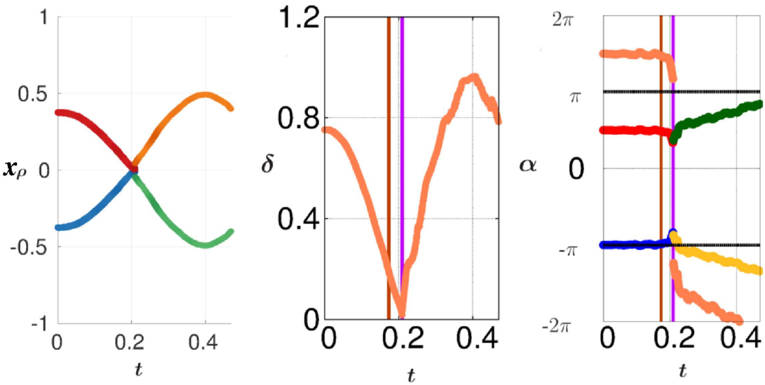


- axial *colliding* motion
- radial drift towards centre

- balance determines regime
- χ unique parameter

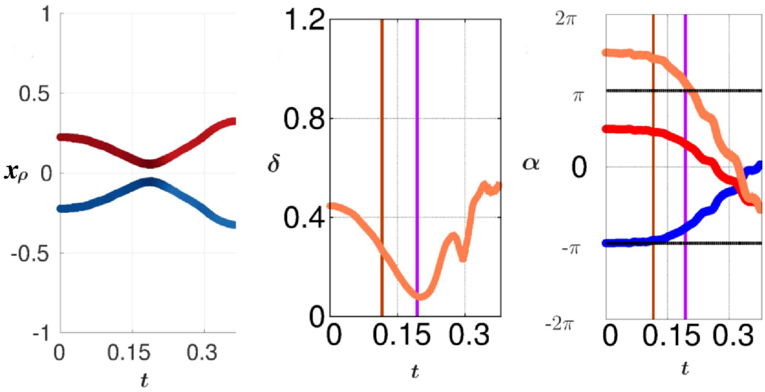
Vortex Interaction Regimes
RECONNECTION

- $\chi = 0.375 \qquad (\chi > 0.3)$



Vortex Interaction Regimes
BOUNCE

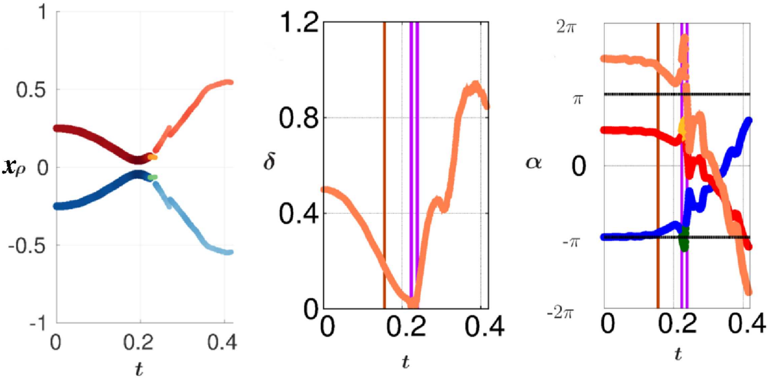
- $\chi = 0.22$ $\quad(\chi < 0.25)$



Vortex Interactions Regimes

DOUBLE RECONNECTION

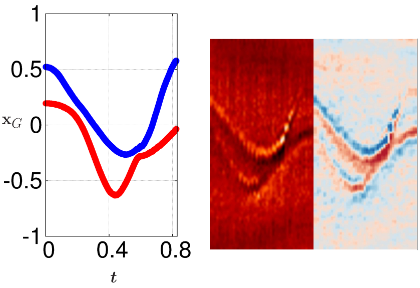
- $\chi = 0.25$ $(0.25 \leq \chi \leq 0.3)$



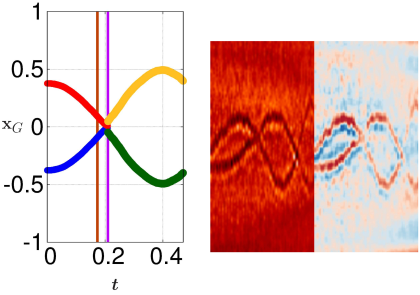
Vortex Interactions Regimes

Numerics vs Experiments

- vortices created via Kibble - Zurek Mechanism
- vortex initial configuration not be predicted
- NEW experiments: axial position, radial vortex orientation



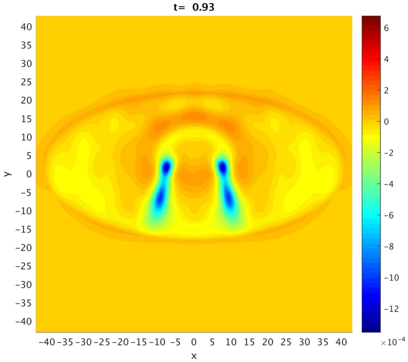
bounce



reconnection

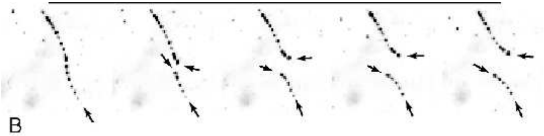
Vortex Reconnections

- $0.35 \leq \chi \leq 0.5$
- $\delta(t) = A\, t^{\alpha}$ scaling
- emission of rarefaction pulse for varying χ



$\delta(t) = A\, t^{\alpha}$ scaling

- Experiments [Paoletti *el al.*, *PNAS* (2008)]

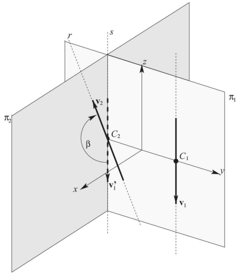


- symmetric
- $\alpha^{\pm} = 1/2$

- Analytics [Nazarenko & West, *J. Low. Temp. Phys.*, **132**, 1 (2003)]
  - density n small
  - drop NL term
  - symmetric
  - $\alpha^{\pm} = 1/2$

$\delta(t) = A\, t^{\alpha}$ scaling

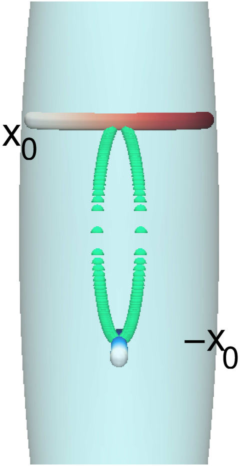
- GPE numerical simulations
  - homogeneous systems
  - *almost* straight vortices
  - fixed $\delta(0) \sim 5\xi$
  - $[A^{\pm}, \alpha^{\pm}] = f(\beta)$



- [Zuccher *et al.*, *Phys Fluids* **24**, 125108 (2012)]
  - $\alpha^{-} \in (0.3, 0.44)$, $\alpha^{+} \in (0.63, 0.73)$, $\alpha^{\pm} = f(\beta)$ asymmetric
  - emission rarefaction pulse
- [Rorai *et al.*, *J. Fluid Mech.* (2016)]
  - $\alpha^{-} = \alpha^{+} = 1/2$ for $\beta = \pi$ symmetric
  - $\alpha^{-} = 1/3\,, \alpha^{+} = 2/3$ for $\beta = \pi/2$ asymmetric
- [Villois *et al.*, *Phys Rev Fluids* **2**, 044701 (2017)]
  - $\alpha^{-} = \alpha^{+} = 1/2$ $\forall \beta$ symmetric $A^{\pm} = f(\beta)$

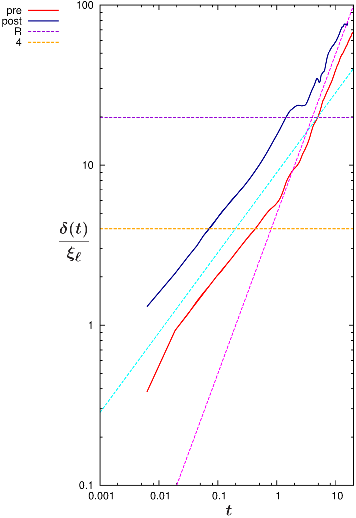
3D Trapped BEC: throwing vortices against each other

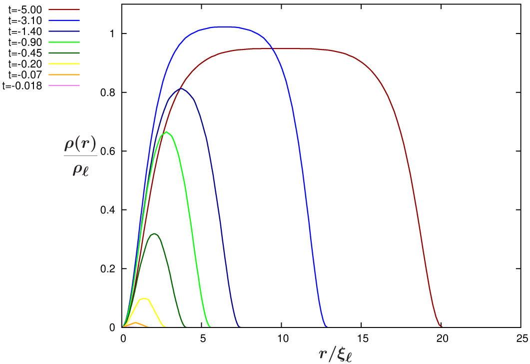
- different $\delta(0) = f(\chi)$
- different $v_x^{rel}(\delta = R_\perp) = f(\chi)$
- reconnections at different $n = f(\chi)$



Reconnections

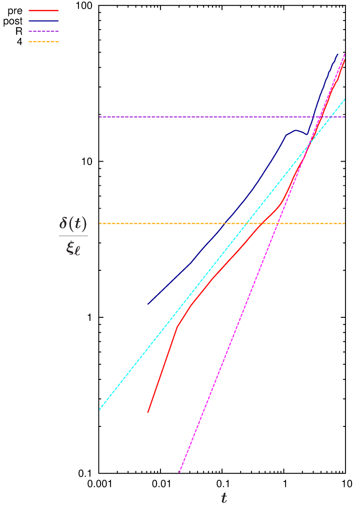
- $\chi = 0.35$

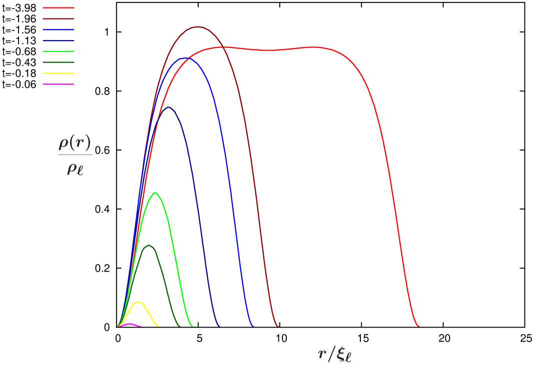




Reconnections

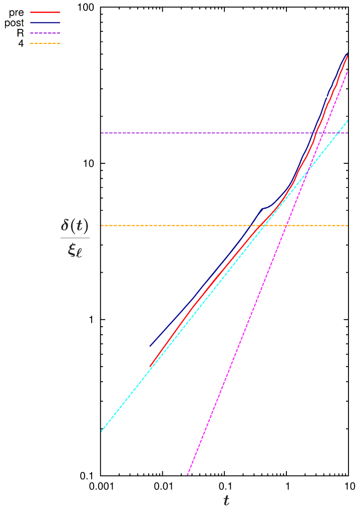
- $\chi = 0.40$

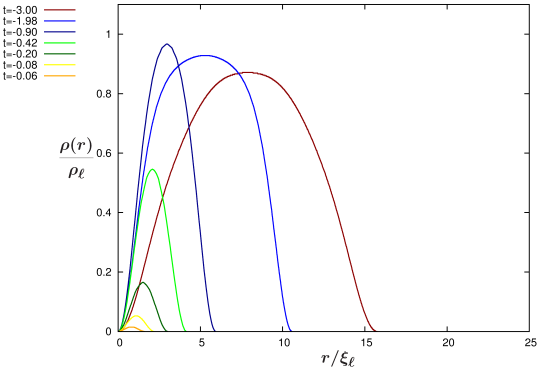




Reconnections

- $\chi = 0.50$

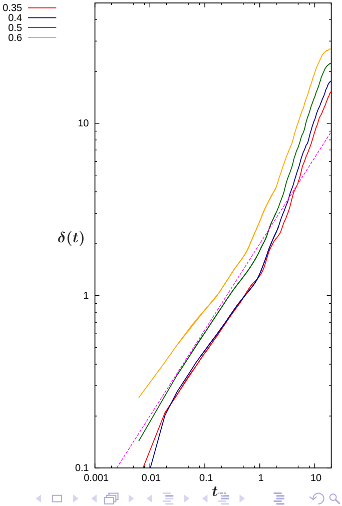




PRE Reconnection scaling

- Inner core region $\delta(t) \lesssim 4\xi$
  - Γ unique paramater
  - $\delta(t) = f(\Gamma, t) = A^{-}_{in}\sqrt{\Gamma t}$
  - $A^{-}_{in} = A^{-}_{in}(\chi)$ $\neq$ HOMOG
  - $t^{*}_{\ell} = t^{*}_{\ell}(\chi) \equiv \dfrac{\xi_\ell}{c_\ell}$

$$= \frac{\hbar}{g n_\ell} = \frac{\hbar}{g n_0 (1 - \chi^2)}$$



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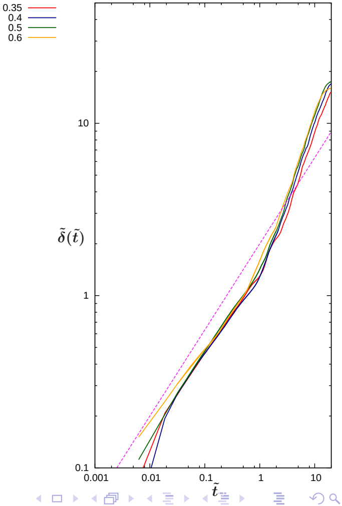
  - $\delta(t) = f(\Gamma, t) = A_{in}^{-}\sqrt{\Gamma t}$
  - $A_{in}^{-} = A_{in}^{-}(\chi) \neq$ HOMOG
  - $t_\ell^* = t_\ell^*(\chi) \equiv \dfrac{\xi_\ell}{c_\ell}$

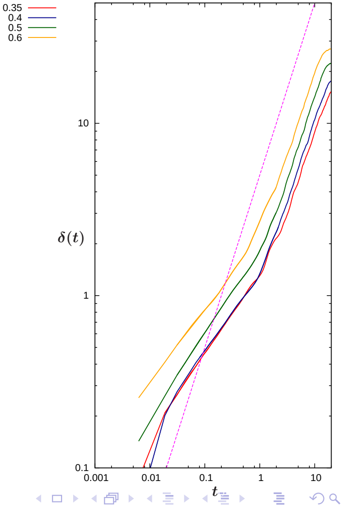
    $$= \frac{\hbar}{g n_\ell} = \frac{\hbar}{g n_0 (1-\chi^2)}$$



PRE Reconnection scaling

- Outer region $\delta(t) \gg 4\xi$
  - dynamics: $\boldsymbol{v}^x_{rel}(\nabla\rho/\rho)$
  - $\delta(t) = f(V,t) = A^-_{out} Vt$
  - $V^*_{orb} = V^*_{orb}(\chi) \equiv C\dfrac{L_{orb}}{T_{orb}}$

    $= C'\dfrac{\chi}{1-\chi^2} R_x\omega_x$
  - $\tilde{\delta} = \delta/L_{orb}$
  - $\tilde{t} = t/T_{orb}$



PRE Reconnection scaling

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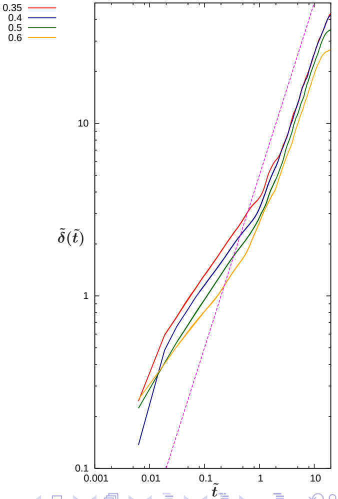


POST Reconnection Dynamics

- post reconnection

  faster than post

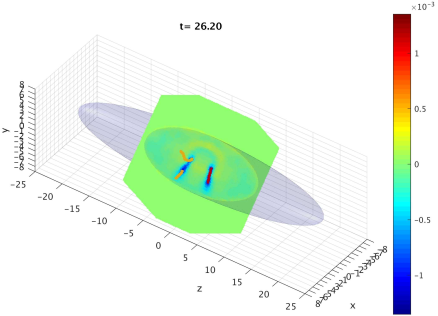
- INHOMOGENEOUS

  $\Delta v \nearrow$ as $\chi \searrow$

- HOMOGENEOUS

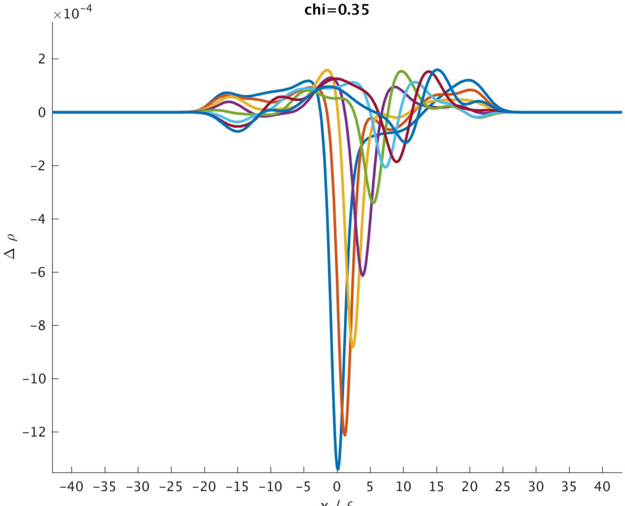
  $\Delta v \nearrow$ as $\perp \rightarrow \parallel$

- density plays fundamental role
- emission rarerfaction pulse
- preliminary results



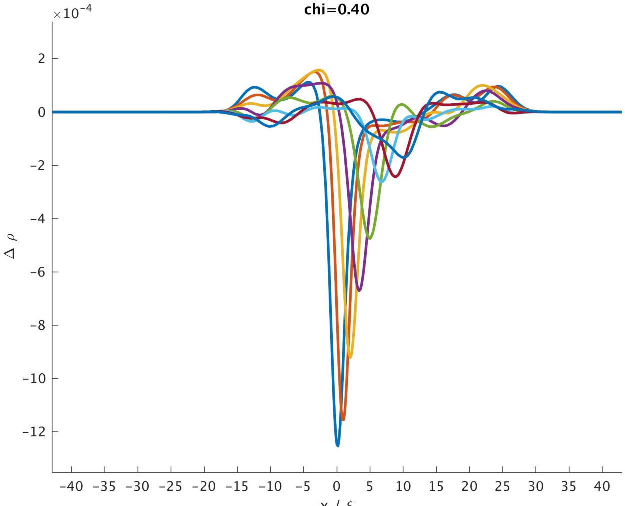
1D Density profiles

- $\chi = 0.35$



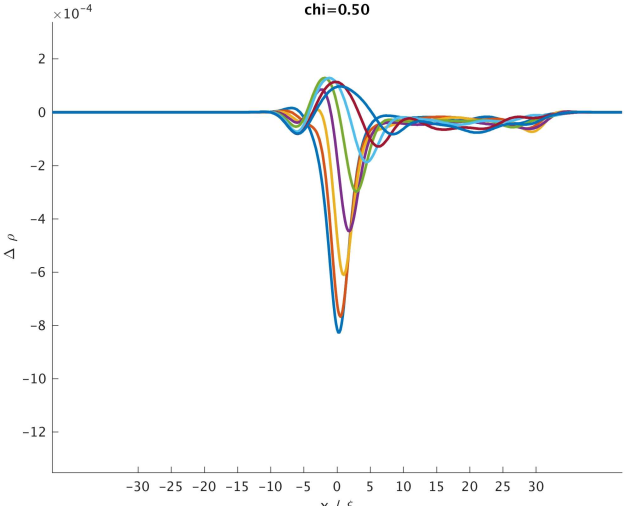
1D Density profiles

- $\chi = 0.4$



1D Density profiles

- $\chi = 0.5$



Summary

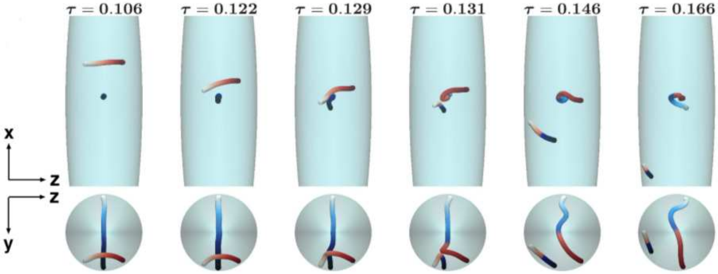
- GPE numerical simulations of vortex dynamics in inhomogeneous confined superfluids
- observed three vortex interaction regimes
  - reconnections
  - bounce
  - double reconnections
  - ejections

  [Serafini, LG, *et al.*, Phys Rev X **7**, 021031 (2017)]
- experimental evidence in Trento
  - orientation of vortices
- Inner core and Outer scalings
- characterization of rarefaction pulse and

  relate to non-symmetrical pre/post behaviour

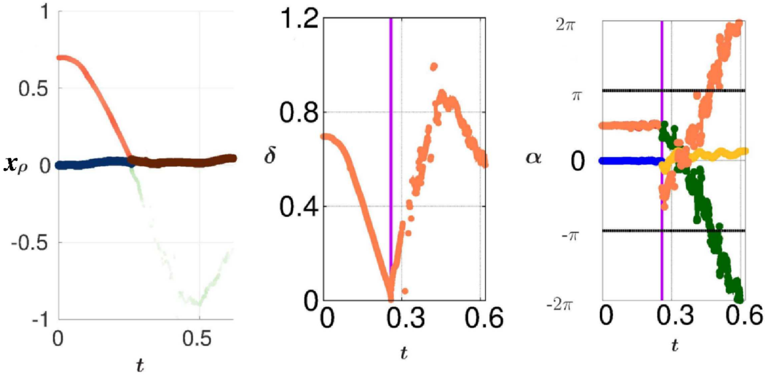
THANK YOU!

Vortex Interaction Regimes
EJECTION

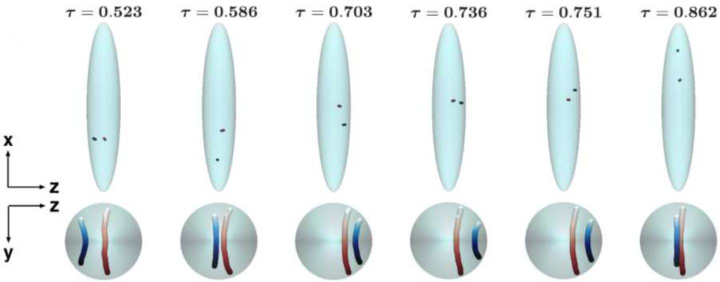


Vortex Interaction Regimes
EJECTION

- $\chi = 0, 0.7$



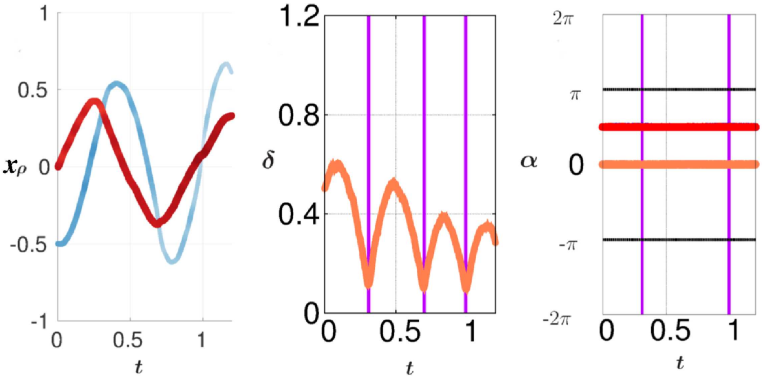
Vortex Interactions Regimes
ORBITING



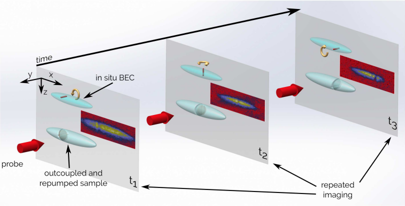
Vortex Interactions Regimes

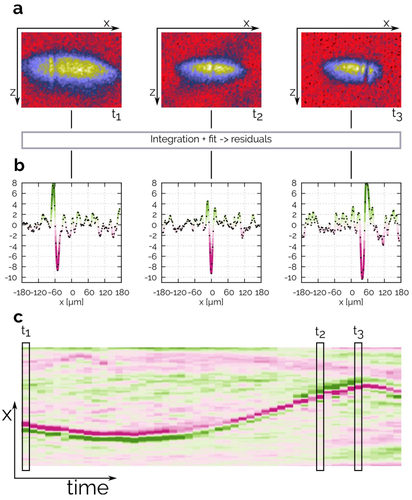
ORBITING

- $\chi = 0.33, 0.5$

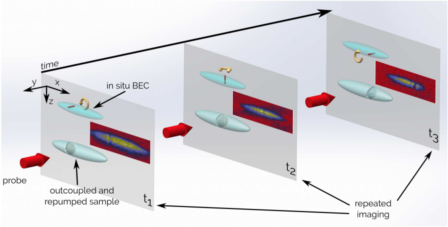


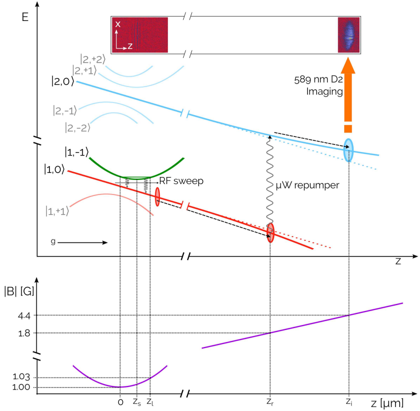
Experiments
Density residuals



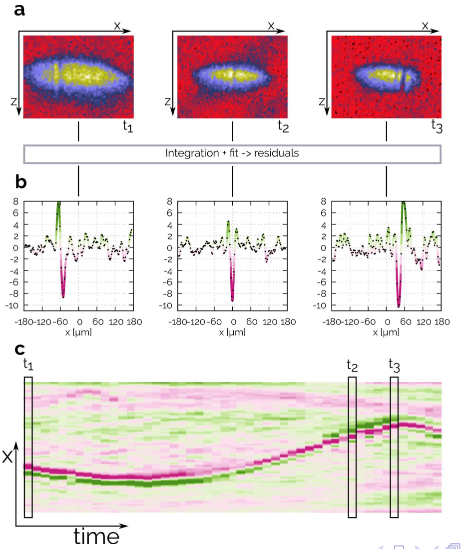


Experiments
Outcoupling

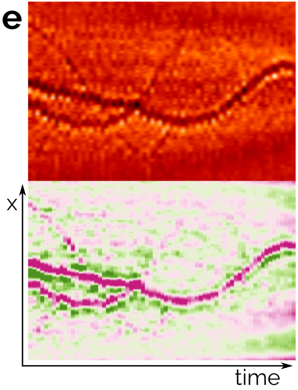




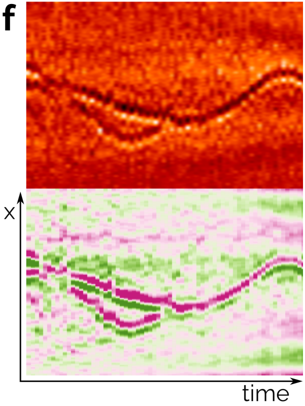
Experiments
Density residuals



Vortex Interactions Regimes
Numerics vs Experiments



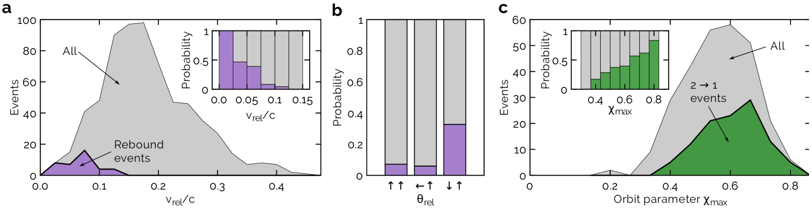
Ejection by reconnection



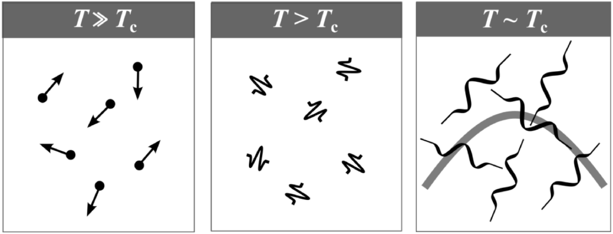
Ejection by orbiting

Vortex Interactions Regimes

Statistics



Bose-Einstein Condensates (BECs)



- $\lambda_{DB} = \dfrac{\hbar}{\sqrt{2\pi m k_B T}}$ $\quad vs \quad$ $d \sim \left(\dfrac{V}{N}\right)^{1/3} = n^{-1/3}$
- $\lambda_{DB}^3 n \sim 1$ onset of condensation T_c
- $T \rightarrow 0$ giant matter wave BEC
- macroscopic wavefuncion $\Psi(\mathbf{x}, t) = \sqrt{n(\mathbf{x}, t)}\mathrm{e}^{\mathrm{i}\theta(\mathbf{x}, t)}$, $||\Psi||^2 = N$
- Gross-Pitaevskii Eq. $\quad \mathrm{i}\hbar\dfrac{\partial\Psi}{\partial t} = -\dfrac{\hbar^2}{2m}\nabla^2\Psi + V(\boldsymbol{x}, t)\Psi + g|\Psi|^2\Psi$

Bose-Einstein Condensates (BECs)

- Gross-Pitaevskii model

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V(\boldsymbol{x},t)\Psi + g|\Psi|^2\Psi$$

- Madelung transformation $\psi = \sqrt{n}e^{i\theta}$, $\mathbf{v} = \frac{\hbar}{m}\nabla\theta$

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = 0$$

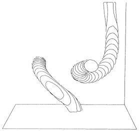
$$mn\left[\frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right] = -\nabla\left(p+p_Q\right) - n\nabla V$$

- $p = \frac{gn^2}{2}$, $p_Q = -\frac{\hbar^2}{4m}n\nabla^2(\ln n)$
- $\Delta \gg \xi = (\hbar^2/mgn)^{1/2}$ recover compressible Euler
- $p_Q \Rightarrow$ reconnections

Quantum Fluids
LENGTHSCALES

$\Delta \lesssim \xi$

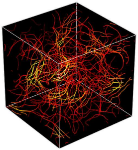
- BECs
- GP model
- vortices, sound waves
- reconnections



[Koplik & Levine (1993)]

$\Delta \gg \xi$

- Helium II
- Vortex filaments
- incompressible Euler fluid
- ad hoc reconnections



[Baggaley *et al.* (2012)]

Classical *vs* Quantum fluids

- Classical Euler (inviscid) fluids

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\,\mathbf{v}\right]=-\nabla p$$

NO reconnections, $\mathscr{H}$ constant

- Classical Navier-Stokes (viscous) fluids

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\,\mathbf{v}\right]=-\nabla p+\eta\nabla^2\mathbf{v}$$

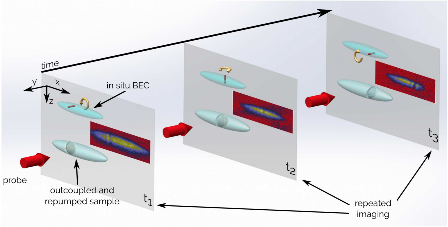
reconnections driven by dissipation, $\mathscr{H}$?

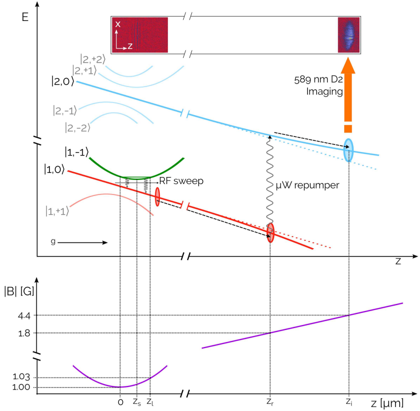
- Quantum BECs

$$mn\left[\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\,\mathbf{v}\right]=-\nabla p-\nabla p_Q$$

reconnections driven by quantum pressure, $\mathscr{H}$?

Experiments
Outcoupling





Numerics VS Experiments

Experiment

- $\omega_\perp = 2\pi 131\text{Hz}$
- $\omega_z = 2\pi 13\text{Hz}$
- $\mu = 10 \div 27\hbar\omega_\perp$
- $T = 200\text{nK}$
- $\dfrac{\Delta N}{N_0} = 0.04 \quad (\mu = \mu(t))$

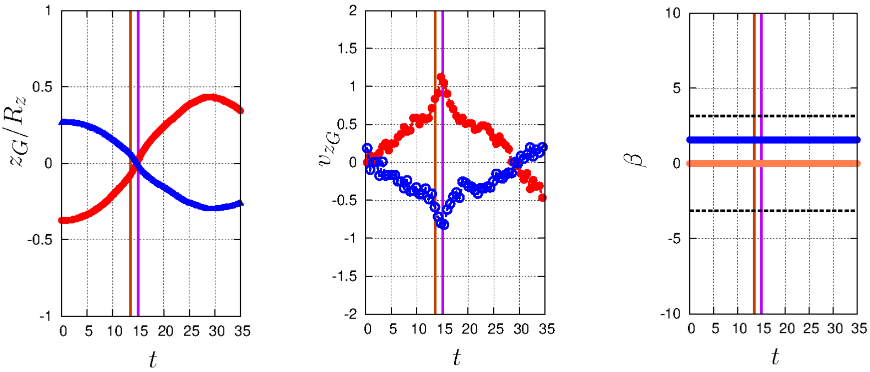
Numerical Simulations

- $\omega_\perp = 2\pi 131\text{Hz}$
- $\omega_z = 2\pi 26\text{Hz}$
- $\mu = 10\hbar\omega_\perp$
- $T = 0K$
- $\dfrac{\Delta N}{N_0} = 0 \quad (\mu = \text{const})$

Vortex Interactions Regimes

Unperturbed dynamics

- $r_{0,y} = \dfrac{1}{4} \quad r_{0,y} = \dfrac{1}{3}$



The "Trento" Experiment

- Creation of topological defects via Kibble–Zurek mechanism
- solitonic vortices: non uniform phase profile

Experiment

- $\omega_\perp = 2\pi 131\text{Hz}$
- $\omega_z = 2\pi 13\text{Hz}$
- $\mu = 10 \div 27\hbar\omega_\perp$
- $T = 200\text{nK}$
- $\dfrac{\Delta N}{N_0} = 0.04 \quad (\mu = \mu(t))$

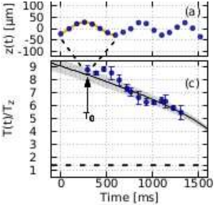
Numerical Simulations

- $\omega_\perp = 2\pi 131\text{Hz}$
- $\omega_z = 2\pi 26\text{Hz}$
- $\mu = 5\hbar\omega_\perp$
- $T = 0K$
- $\dfrac{\Delta N}{N_0} = 0 \quad (\mu = \text{const})$

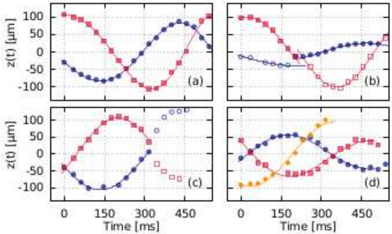
The "Trento" Experiment

- One vortex dynamics

$$T = \frac{8\pi(1-r_0^2)\mu}{3\hbar\omega_\perp \ln(R_\perp/\xi)\omega_z}$$



- Two vortex dynamics



GPE simulations

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\nabla^2\psi + \frac{1}{2}\left(x^2 + y^2 + (\omega_z/\omega_\perp)^2 z^2\right)\psi + 4\pi\left(\frac{Na_s}{\ell}\right)|\psi|^2\psi$$

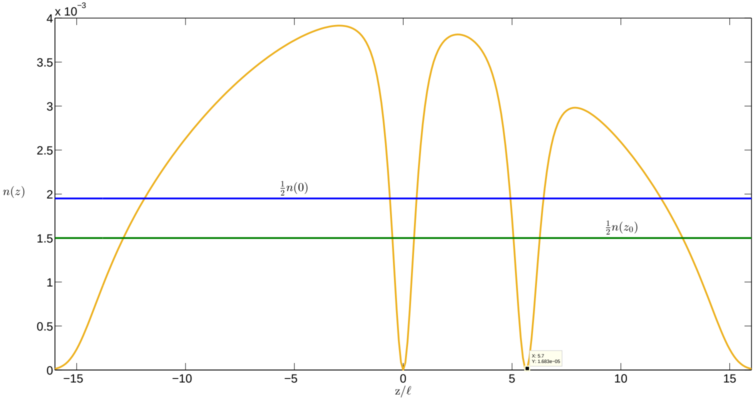
- $\ell = \sqrt{\dfrac{\hbar}{m\omega_\perp}}$
- $\tau = \dfrac{1}{\omega_\perp}$
- $\epsilon = \hbar\omega_\perp$

GPE simulations

Parameters

- second–order finite difference schemes
- 4^{th} order Runhe-Kutta time integration
- OpenMP parallelization
- $\Delta x = \xi/3 = 0.1$
- $\frac{\Delta}{t} < \frac{1}{2}(\Delta x)^2 = 1,25 \times 10^{-3}$
- $R_\perp = 3.16, \quad R_z = 16$
- $NX = NY = N_\perp = 160\ , \quad N_z = N_\parallel = 480$
- $N_t = 2 \div 5 \times 10^4,\ T \sim 0.5d$
- 300 MB / dumped wavefunction

Density profiles



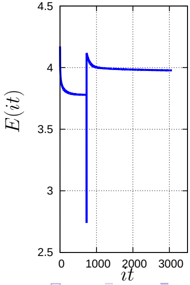
$$\xi = \frac{\hbar}{\sqrt{2mgn(z)}} \Rightarrow a_0 = 12 \div 15 \Delta x$$

ψ Relaxation - Imaginary time advancement

- ψ_0: Thomas – Fermi profile

$$\psi_0 = \begin{cases} \dfrac{\mu - V}{g} & \text{for} \quad \mu \geq V(\mathbf{r}) \\ 0 & \text{elsewhere} \end{cases} \qquad \frac{x^2}{R_\perp^2} + \frac{y^2}{R_\perp^2} + \frac{z^2}{R_z^2} \leq 1$$

- τ_0 via Energy condition: $\dfrac{\Delta E}{E} < 10^{-6}$
- vortex imprinting $\psi = \psi_0(\tau_0)\Pi_i\psi_i$
- τ_v same Energy condition $\dfrac{\Delta E}{E} < 10^{-6}$



Vortex identification

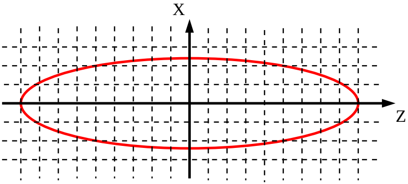
Pseudo–vorticity field

- $\widehat{\omega}$
- $\rho = 0 \;\; d\rho = 0$
- $d\psi_r = d\psi_i = 0$
- $\nabla\psi_r \cdot \widehat{\omega} = \nabla\psi_i \cdot \widehat{\omega} = 0$
- $\widehat{\omega} = \dfrac{\nabla\psi_r \times \nabla\psi_i}{|\nabla\psi_r \times \nabla\psi_i|}$
- $\widehat{\omega} = \dfrac{\nabla\rho \times \nabla\theta}{|\nabla\rho \times \nabla\theta|}$

Difficulties

- $\rho(\mathbf{r}) \to 0$ as we move to the boundaries
- vast region with $\rho(\mathbf{r}) \to 0$ when reconnections
- FIRST POINT NECESSARY

Vortex identification



- Γ on each mesh element
- node with the lowest density ρ_{min}
- multiple reconstruction of same vortices
- potentially 12 equivalent reconstructions
- improved with sub grid resolution